# Teacher Learning and Pedagogical Shifts Subsequent to Professional Development Experiences

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## ABSTRACT

*Mathematics pedagogy is a complex and multilayered practice, a practice that is formidably difficult to change. The authors of this paper were interested in understanding the changes that teachers from the Secondary Numeracy Project (SNP) have made to their practice. The paper focuses on a case study of one teacher, looking at how her teaching is changing in the year after professional development experiences of SNP. Teacher learning and associated changes in practice are seen as occurring across a continuum; the case study highlights that, for this teacher, participation in SNP was an impetus for her to continue to learn through inquiry into her own practice.*

## INTRODUCTION

Promoting teacher learning and development in ways that enhance students' learning is a central object of any professional development. In mathematics education, current system-wide numeracy initiatives – the primary-based Numeracy Development Project (NDP) and the Secondary Numeracy Project (SNP) – aim to develop teacher knowledge and raise student achievement. These projects advocate reforms that demand a major shift in teachers' thinking and practice. Given that we know that mathematics pedagogy is a complex and multilayered practice (Anthony & Walshaw, 2007), a practice that is formidably difficult to change (Cobb, McClain, de Silva Lamberg, & Dean, 2003; Little, 2003; Spillane, 2000), we were interested in understanding the changes that teachers from the SNP project have made to their practice. Using a case study of one teacher, we look at how her teaching is changing in the year after her professional development experiences. We take the position that teacher learning and associated changes in practice occur across a continuum and thus are interested to explore how the teacher might continue to learn through inquiry into her own practice.

The content of the SNP school-based professional development programme shares features with the established Numeracy Development Project implemented within primary schools. The focus is on a Number Framework detailing progressions of number knowledge and calculation strategies, together with diagnostic interviewing. The aim of the professional development is to support teachers in developing students' numerical competency and understanding and generalisation of number strategies as a basis for algebraic thinking (Hannah, 2006). The advocated pedagogical approach[1] is based on Skemp's (1976) theory of relational understanding and its derivative practice of students' strategy sharing. It is structured around a formalised model of developmental stages of number understanding. Learning experiences transition through physical representations, imaging, towards abstract mathematical concepts and algebraic thinking (see Figure 1). The overarching aim is to support students to develop the skills and dispositions towards a flexible understanding of how numbers work and increasingly sophisticated ways of thinking and reasoning.

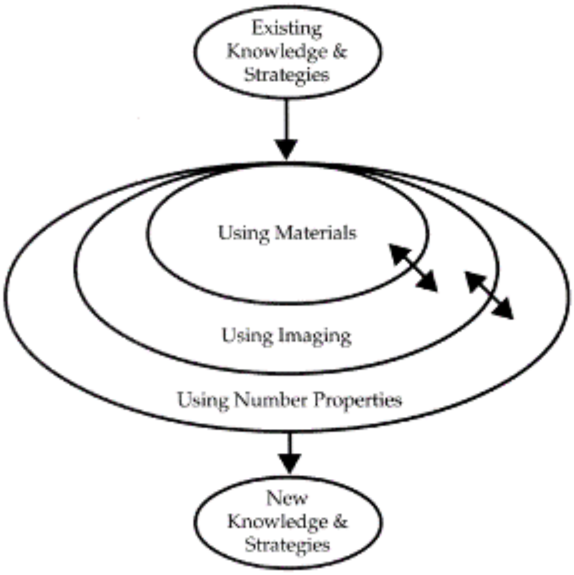


**Figure 1.** The Main Phases of the Strategy Teaching Model (Hughes, 2002)

To date, evaluations offer promising signposts that SNP project has made a positive difference for teachers and their students. Evaluations of the SNP using self-reported questionnaire data suggest that the majority of teachers report growth in their pedagogical content knowledge and mathematics teaching practices in years 9 and 10 (Harvey & Higgins, 2007). A more recent analysis by Harvey and Averill (2009) of self-report data from teachers from four 'successful' SNP schools suggest that changes associated with SNP are very individual: some teachers felt they had undergone major changes of approach, while others had been more cautious and had adopted relatively few changes. Questionnaire responses included references to an increased range of teaching strategies – including increased focus on student thinking and students explaining their thinking; increased focus on developing and assessing students' mathematical understanding; and increased use of real-world contexts.

[1] See www.nzmaths.co.nz/teaching-numeracy for further information about SNP

## INSIDE THE CLASSROOM

In this paper we report on a case study involving one teacher – identified by the pseudonym Amy. Amy, an experienced, well-regarded teacher participated in significant professional development opportunities within the SNP during 2006. In 2007 we observed a sequence of ten lessons on fractions with Amy's Year 9 class of high achievement-grouped students. As part of a larger *Teaching and Learning Research Initiative* study examining the teaching and learning nexus, data were generated from observations, video records of lessons, and a series of stimulated recall interviews with both teacher and students. In this paper we use this data to explore how teacher learning from the SNP plays out within Amy's classroom, looking specifically at how the SNP project is making a difference to the ways she is teaching and thinking about learning for her students.

In looking at Amy's pedagogical practices we considered first the indicators of 'why' she wanted to make changes in her teaching of fractions subsequent to her involvement in the SNP professional development programme. In claiming that she taught the unit differently, 'I mean I totally changed what I would have normally have done for year 9', Amy noted that she 'used to focus on what to do rather than the understanding … ordering, equivalent fractions, different representation plus the algorithm … but [I now realise that] understanding of concepts are important'. Alongside a strong belief that a focus on understanding was crucial, Amy noted that the introduction of the SNP diagnostic assessment material provided her with evidence of her students' current levels of understanding – understanding that she felt she had previously taken for granted: 'I knew fractions were difficult for students but I hadn't realised quite what was so difficult … when you look at it more, you realise just how complex fractions are'. In addition, Amy was able to clearly identify that she had strengthened her pedagogical content knowledge – specifically with regard to fraction representations and potential areas of students' difficulties. The SNP also challenged Amy's thinking about participatory practices within her classroom, with Amy noting that sharing strategies within discussion was something she was keen to develop more fully. Practices of grouping students according to numeracy stages, or extensive use of group work, was not at this point of time considered a priority within her classroom.

## A FOCUS ON UNDERSTANDING

Whilst it is apparent that there are a number of factors prompting Amy to reconsider her practice we want to focus within this paper on what appears to have been an over-arching goal for Amy – that of developing understanding. To support students' development of understanding, specific changes in emphasis evident in Amy's practice included:

- the establishment of a norm that understanding was a desired learning outcome;
- the selection of tasks designed to build on students' existing knowledge and ways of thinking;
- the use of concrete materials and a range of representations; and
- a focus on collective discourse centred on students' thinking.

Amy's reference to the 'what to do' versus 'understanding', as noted above, inferred that she was aware of the distinction that is often made between forms of knowledge associated with procedures or techniques and knowledge concerned with conceptual or relational understanding (Skemp, 1976). For Amy, the goal of understanding was explicitly and frequently shared with her students:

> You know the whole numeracy is going for understanding rather than a rule. (L8)

In order to highlight understanding as a goal there were instances where Amy appeared to be *imploring* students to understand, or *demonstrating* how in fact she herself was understanding:

> I want to show you a way of multiplying using area and I want to show you the understanding or the reason of what you are doing rather than just the rule. (L7)
>
> That's where this method falls apart with the subtraction. That's what I wanted you to find out. (L9)

Although directives such as these may highlight the importance of understanding and orientate students to focus on particular aspects, they will not in themselves occasion student understanding.

To further support the development of understanding, Amy was deliberate in her choice of tasks; she selected tasks designed to prompt activities that would promote relational rather than procedural understanding. She moved from exclusive use of a textbook to the use of a wider range of instructional tasks – including the use of manipulatives and folding and cutting activities. For example, in exploring the equivalence concept, students constructed fraction fringes that enabled the physical identification of a range of equivalent fractions using area models, and in introducing multiplication students used arrays to solve 1/2 of 1/3 as represented by the intersecting area. Reflecting on these changes, Amy noted:

> It's not that I didn't use diagrams before … I would use them to illustrate an example in notes, say … but now I use them to develop the understanding, see in different ways, so they can get the picture then move onto the abstract. (Post unit interview)

Significantly, Amy noted that she had not previously used tasks that involved the use of concrete material with her high-achieving students. Promoted as a tool, diagrams provided thinking spaces to help students organise their mathematical thinking. For example, in a discussion of the calculation involving converting 3 and 2/3 to an improper fraction, Amy justified the method by asking students to:

> Imagine how many one-thirds are going to be in each whole … and how many are going to be left over in the extra diagram. (L3)

In addition to inviting students to use diagrams, Amy frequently affirmed the relevance and utility of diagrams both for students' thinking and her own thinking:

> Yes, when you are halving you are doubling that bottom number. Nice pattern I quite like that it is quite nice to see that on the grid. (L4)
>
> Did it help drawing a diagram? I find it helps to see. (L8)
>
> What I wanted you to do was to see visually what you were doing. (L7)

Amy reiterated her purpose for linking the diagram and the calculation – the intent was that students would develop understanding of how the computation algorithms worked through representing the process first with diagrams:

> What we are going to do now is go from drawing diagrams to coming up with a method so that we don't have to draw the diagram every single time. (L3)
>
> So what I have tried to get you to do is develop a method knowing and understanding what it is you are doing. (L3)

Within the series of lessons on fractions, a key feature of developing student understanding was supporting students to 'come up with a rule'. As we have seen, one way Amy organised this was to encourage students to symbolize diagrammatic solution strategies. For example, when using overlapping tiles to represent multiplication of fractions Amy told her class to:

> Look at it through a diagram and I want to see if you can come up with the rule … did you notice the pattern in all of the questions that you did? Rather than me coming in and going here's the rule, I want you to see what you were doing in terms of area. (L7)

Building on students' informal strategies was another way to develop understanding. Starter activities were used to invoke students' informal solution strategies that were then used as a basis for deriving the more formal rule. For example, when teaching division of fractions, in contrast to her previous provision of the invert and multiply algorithm, Amy's intent was to invite her students to 'come up with a rule' based on their solution methods to a set of word problems. Students first completed a three problem set involving division scenarios such as:

> *The Murphy family had a party and had 3 and 3/4 pizzas left over. The kids decided they could invite friends over the next day for pizza. They figured each friend would eat 3/8 of a pizza. How many people could be fed?*

In this lesson we observed the majority of students solved this problem by re-presenting the pizza problem as 30/8 ÷ 3/8. This then was solved as equal to 10/1 = 10 people or (less frequently) by using repeated subtraction. The algorithm that represents the procedural reasoning in this type of division is the common-denominator algorithm for the division of fractions:

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd} = \frac{ad}{bc}$$

However, linking students' informal strategies to formal algorithms is not always a straightforward process. In this instance the shared strategies that students presented to the whole class were unable to be linked to the 'formal' method involving reciprocals that Amy was about to present as 'notes'. Such a link, while arguably quite challenging to make, would more easily be made by starting with examples that involve whole numbers divided by a fraction (see Sinicrope, Mick, & Kolb, 2002). Without the connection between the informal and formal solution strategies, it was not surprising to find that both of the students who were interviewed post lesson expressed uncertainty as to why division problems 'turned into multiplication' problems. The difficulty was highlighted by one student who demonstrated to us that 2/3 ÷ 3/4 = 3/2 × 4/3 = 12/6 = 2. The student's uncertainty was captured in her reflection on the flip and multiply explanations provided to accompany the 'notes' part of the lesson:

> I always thought the rule was, 'If you change one thing about a fraction you have to change something about the other one like to make it so it's fair'… I was a little confused thinking well that's just changed everything I ever thought about fractions. (Post lesson 8)

In addition to moving from the informal to the formal strategies as a way of developing understanding of where 'rules' come from, Amy also indicated to students that having multiple ways, or preferred ways, to solve problems was a desirable feature of mathematical sense-making. In asking students to share their solution strategies, Amy frequently prompted students to offer alternatives and highlighted that problems could often be solved in a variety of ways:

> You can use your fraction fringes, but don't have to. (L6)
>
> There are many different ways to get the answer. (L6)

## GOOD THINGS TAKE TIME: LEARNING ACROSS THE CONTINUUM

Teacher learning associated with professional development programmes is not and should not be viewed as a 'fixed' entity. Effective professional development requires that teacher learning be self-sustaining and generative (Franke & Kazemi, 2001). Thus, learning and making changes to one's practice will occur along a continuum (Feiman-Nemser, 2001). For Amy, it is clear that her focus on the development of students' understanding involved a process of ongoing inquiry and resultant shifts or adaptations of her pedagogical practice.

Reflecting on the lesson sequence, Amy noted that a big change in her classroom was 'how much more the students were thinking'. With reference to an episode in which a student questioned why 'when multiplying fractions the answer is smaller', Amy expressed the need to learn more about how to attend to students' thinking on-the-spot. In this instance, her response – 'I'm impressed with you coming up with that because whenever you multiply you think it's going to be bigger … we saw that in decimals as well didn't we?' – commended the student's observation and attempted to make connections with decimals. However, without further interaction with the student Amy's response was unlikely to significantly extend the student's mathematical thinking. Amy noted that her resistance to further engage with the student was balanced with the need to attend to other students' requests for help.

Another area of development for Amy was the social process of negotiating students' meanings. A teacher must be able to differentiate between the mathematical integrity of students' ideas, and be able to synthesise students' individual and sometimes disparate contributions (Walshaw & Anthony, 2008). Amy, on several occasions found herself faced with situations that required careful unpacking of students' responses, alongside the need to respect occasional more intriguing or obscure contributions. During post-lesson interviews Amy commented on several proffered solution strategies. For example, in Lesson 6 Amy noted her surprise with a student's solution strategy to the problem: _ + 2/3 = 1 11/30. The student offered the following explanation:

> I went that the answer was like 41/30. I knew that 10 times 3 was 30 so that the denominator on the one that you can't see was 10. And then I knew that 10 times 2 was 20 so I had to make 21 and 3 times 7. (L5)

Amy reflected:

> She worked out that the first fraction had to be over 10. Her explanation surprised me – it was quite interesting. It was quite a hard way … That was really a cool way of doing it. It was quite incredible … I don't know whether the rest of the class would have clicked onto what she was doing. It might have been a few of the girls in the class would have worked out what she was doing but not many … I would have liked to have unpacked that further to explain to the class. Sometimes it's so complicated and it might be unique to that student and maybe the rest of the class aren't going to pick up on that and so – and also sometimes they come up with examples that might only work for that particular one, they may not work for every single question.

Responding in the instance, Amy paralleled this method to using the inverse operation of subtraction to find a missing number discussion how to solve _ + 3 = 13 by using an 'undoing' strategy. She did not, however, directly address the addition of fractions algorithm that had been offered by the student. Of interest here is the description by Ainley and Luntley (2007) of *attention-*

*dependent knowledge* – knowledge that enables teachers to respond effectively to what happens during the lesson. These researchers define attention-dependent knowledge as:

> A part of what experienced teachers know, both in the sense that they have attention skills which enable them to 'read' the activity of the classroom, and that they use the knowledge they gain by and from this attention in making judgments about how to act. (p.1137)

Amy's attention appears to be consistent with a focus on what Ainley and Luntley label *cognitive problems,* where students show differing understanding of mathematical ideas promoted by the teacher, rather than a focus on *cognitive opportunities,* opportunities that involve trying to extend students' thinking and understanding. Amy noted that only a few students in the class 'would have clicked onto what [the student] was doing"' – for that reason she was hesitant to unpack it any further. In continuing to develop her practice Amy is aware that expecting her students to participate more fully as mathematical thinkers – rather than just doers – will require her to gain more expertise in managing unexpected student responses and questions.

In developing structured mathematical knowledge and understanding, pedagogical practice also needs to support students to arbitrate and make mathematical connections between alternative solution strategies. However, occasionally it was evident that students were unsure as to why alternative strategies were offered. For example, when discussing the multiplication of fractions the teacher-provided notes detailed two ways of proceeding – multiplying across numerator and denominators and then simplifying to an equivalent fraction, or cancelling first and then multiplying across numerators and denominators. There was no mathematical discussion as to the connections or relative efficiencies of these two 'methods'. Presented as an 'either or' based on personal preference, Amy reflected in post-lesson interviews that 'In the past I've found that some students like to cancel or simplify at the end and some students like to do it beforehand so I just wanted to show them the two different scenarios they could use'.

In reflecting on her students' learning at the end of the lesson sequence Amy noted that:

> I think they are still struggling with multiplying and dividing. I think their understanding is getting there, I wouldn't say that it's absolutely fantastic but I think I've made a little bit of difference to get them there.

Cognisant that her learning as a teacher is ongoing, Amy indicated that she was already thinking about what to do differently next time she taught fractions:

> I think next time I would definitely introduce the adding, and the multiplying and dividing how I did this time with giving them some concepts in the concrete and the playing. I would probably spend less time with this particular class on the equivalent and the simplifying. I would probably go through that a lot quicker. The fringes I'm not quite sure, I thought it was quite a nice intro, maybe some of the learning was in the cutting up.

## CONCLUSION

Amy – an experienced and highly regarded teacher – was in no doubt that the SNP professional development experience created a stimulus for shifting her pedagogical practices. Focused on supporting students' emergent understanding of fractions, specific shifts in her pedagogical practice included an increased use of concrete materials, a greater acknowledgement of students' existing knowledge, the valuing of multiple solution strategies, and an associated press for understanding. Alongside this, Amy was acutely aware that the nature of the tasks and tools could influence students' opportunities to develop understanding. Her move to incorporate a wider range of tools to support student thinking, her focus on multiple representations, and the utilisation of tasks that encouraged students to draw on their existing knowledge, all signify overt shifts in her practice.

Removed from the professional development support of SNP, Amy is now faced with the challenge of attending both to her students' and her own learning needs. How teachers continue to learn from interacting and observing with students in their own classroom has only recently been the focus of research (e.g., Davies & Walker, 2007; Margolinas, Coulange, & Bessot, 2005). Reflecting on lessons episodes (both on-the-spot and retrospectively) is one way to think and learn more about students' ways of thinking. Another way is for teachers to interact with students in different ways – not only listening to students' thinking, but also deliberately asking questions that clarify their own understanding about students' thinking (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). In learning more about students' thinking, we hypothesise that Amy's attention as an expert teacher will move away from the more traditional focus on cognitive problems (Ainley & Luntley, 2007) to include a greater focus on cognitive opportunities. The extent to which Amy is able to extend her attention-dependent knowledge will depend on whether she continues to engage in practical inquiry within her own classroom. Amy's capacity to engage in practical inquiry, in combination with the support of her professional community, will be a significant factor in the generative development of numeracy practices advocated in the SNP aimed to further develop students' mathematical thinking and understanding. The conditions that increase the likelihood of this occurring are worthy of further investigation.

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