Responsive Teaching in Primary Mathematics: Linking current theory to practice and planning

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ABSTRACT

Facilitating a more responsive style of teaching in primary mathematics has implications for not only teaching practice, but also for how we plan for our learners. Grouping decisions, task selection and teacher confidence in mathematical content are important considerations when developing inquiring mathematical communities. This article introduces some core elements of current theory for primary teachers who want to include inquiry principles in their mathematics programme and increase their confidence levels when teaching primary mathematics. Teaching practices that promote learner agency are identified, followed by a planning process that allows for the flexibility required for responsive teaching and also encourages teachers to critically engage in mathematical content as they plan for their learners.

INTRODUCTION

Primary teaching has developed to respond to children's mathematical thinking, de-emphasising the role of speed and memorised procedures, while facilitating mathematical thinking through guided inquiry. Guided inquiry in this sense, refers to teacher engineered inquiry that also involves explicit acts of teaching (Fry & Hillman, 2018). Guided inquiry in mathematics reflects current theory about how children best learn mathematics, and includes a growing emphasis on learner agency and self-efficacy. Learner agency refers to learners having some say in their learning, resulting in a shift of the power balance between learners and teachers which includes the concept of children being active participants in their learning as opposed to passive recipients of information (Fraser, 2016). Self-efficacy refers to learners' self-belief and confidence in themselves as mathematical thinkers (Bandura, 2007; Zimmerman, 2000).

In traditional primary mathematics classes, children who got the most answers correct in the shortest space of time were considered 'capable mathematicians' (Boaler, 2016). While the work of Jo Boaler has contributed to de-emphasising the role of speed and answer driven approaches to problem solving, current research on best practice also emphasises mixed ability groupings, task selection and the role of questioning (Boaler, 2016; Boaler & Brodie, 2004; Hunter, 2010; Sullivan & Lilburn, 2017). With the current emphasis on guided inquiry in mathematics, traditional planning methods do not facilitate the responsive nature of teaching in this way.

The initial impetus to design guidelines for a planning process for guided inquiry in mathematics came from the need to support final year pre-service teachers (PSTs) to plan for responsive teaching. The main goal was to help PSTs avoid the act of 'telling' when learners became unsure of mathematical concepts and to encourage 'productive struggle' through planned questions and prompts in the planning phase of preparing for teaching (Ingram et al., 2019, Sullivan et al., 2012; 2018; Sullivan & Lilburn, 2017.) The process for planning for responsive teaching in mathematics was also trialled with five teachers in a primary school setting that emphasised tasks developed over several mathematics lessons and progressed in response to learners engagement and understanding[1]. Before introducing considerations of planning for responsive teaching, however, some underlying concepts will be explored.

Though there are many complexities in the primary mathematics classroom, I have isolated three concepts I consider critical to enhancing student learning opportunities: responsive teaching through guided inquiry, grouping, and task selection. I summarise the identified concepts, then I offer suggestions for a planning process that encompasses the belief that planning "is pivotal to student learning…teachers can enhance or constrain student learning [through planning]…" (Rubie-Davis et al., 2016, p. 136). Furthermore, I argue the *process* of planning can support teachers to teach confidently and responsively in mathematics.

INQUIRY IN MATHEMATICS

The roles of the teacher and learners in a mathematical inquiry classroom are radically different to traditional mathematics instruction (Anthony et al., 2015; Boaler, 2016; Hunter 2010). Traditional, cognitive views of learning emphasise the acquisition of knowledge, with the teacher being the transmitter of information and procedures, and the learner being a receiver and processor of information (Sewell & St George, 2016). In this scenario, the teacher holds the power and directs the learning. Conversely, when developing learner agency, "both teachers and students must be able to contribute their experiences to engage in genuine inquiry...as each member builds on the understanding of others, including the teacher, new learning is co-constructed" (Sewell & St George, 2016, p. 244). This means the teacher must embrace a new balance of power within teaching and learning. Another contributing factor to successful guided inquiry in mathematics is the level of mathematical content knowledge of the teacher (Boaler 2008; 2016; Fraser, 2016; Hunter, 2010). A guided inquiry approach to teaching primary mathematics requires the teacher to have an in-depth understanding of the mathematics they are looking and listening for as learners work their way through tasks.

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[1] Currently unpublished data

In-depth understanding empowers teachers to recognise and engage in 'teachable moments' and to distinguish between opportunities for explicit acts of teaching and times when a well-placed question might encourage deeper and independent thought by learners. Identifying when to explicitly teach and when to use questions to encourage learners to think critically, requires the teacher to be acutely aware of the potential mathematical content of the rich task (Boaler, 2016; Boaler & Brodie, 2004; Sullivan et al., 2012). To facilitate a balance between learner agency and developing mathematical understanding, teachers shift between explaining concepts to learners, and encouraging mathematical discussion through carefully worded and timed questions that challenge learners to justify, represent or alter their thinking (Boaler & Brodie, 2004; Fry & Hillman, 2018; Sullivan 2018). Sullivan (2018) refers to 'enabling prompts' that can vary an aspect of the task, and teachers include this idea in our concept of scaffolding questions in planning for a learning task. How learners respond to tasks, to some extent, will depend on how they are grouped for instruction.

Grouping

Traditionally, teachers assigned learners to ability groups believing that this allowed them to plan meaningful learning experiences for a specific group of learners. Research not only refutes the belief that ability grouping benefits learners, but studies conclude placing learners in groups based on ability can actually hinder the progress of some learners (Anthony & Hunter, 2017). For example, Rubie-Davies (2015) found that teachers who had fixed notions of ability were more likely to use and to set tasks for lower achieving students that did not provide opportunity for higher order thinking. Studies show that learners who work collaboratively with peers of varying abilities are exposed to different levels of mathematical thinking, leading to higher achievement levels than attained by those who work in groups of similar ability (Anthony & Hunter 2016; Boaler, 2008; Jorgensen & Dole, 2011; Rubie-Davies, 2016). Flexible, heterogeneous grouping, paired with skill-based workshops in response to student need, requires the teacher to believe that all students can learn mathematics to a high level. Anthony and Hunter (2017) explain this belief as "the view that mathematics learning is a process determined by effort, rather than fixed notions of ability" (p. 76). The idea of effort being a pre-requisite to learning underpins Dweck's (2006) growth mindset theory and is the foundation of Boaler's Mathematical Mindset series (Boaler, 2016; Boaler et al., 2017; 2018a; 2018b). For mixed ability grouping to be successful, selecting appropriate tasks for learning experiences is critical.

Tasks

Tasks deemed suitable for rich learning experiences can be referred to by a variety of terms: rich tasks, investigations, problem solving, open ended tasks, inquiry tasks, good tasks, worthwhile tasks and challenging tasks. (Boaler, 2016; Jorgensen & Dole, 2011; Sullivan & Lilburn, 2017). In this article, I use the term, 'rich tasks', which are defined as those that meet most, but not necessarily all, the criteria listed in Table 1.

Table 1.
Criteria for Rich Tasks

Criteria	Implications
Low entry - high ceiling	Refers to tasks that have multiple entry levels and lend themselves to further investigation. All learners can be challenged by the task. All learners can attempt the task at some level before explicit teaching is enacted.
More than one approach	Learner agency is increased by learners being able to choose how they approach a problem and what strategies they apply.
May have more than one correct answer	Tasks put the onus of proof on learners and encourage learners to come up with more than one solution they can prove is accurate.
Requires learners to justify/explain/prove their solution in a variety of ways	Tasks lend themselves to multiple representations of process and solution. Learners value the importance of mathematical thinking and see maths as more than just seeking a correct answer.
Includes opportunities for collaboration amongst learners	Collaboration and discussion amongst learners are inherent in tasks which expose learners to different ways of thinking mathematically.
Requires learners to have a growth mindset	This includes learners accepting challenge with the belief that being challenged means they are learning. The concepts of growth mindset can be taught alongside the maths.
May take several sessions	The tasks allow for different approaches which can be explored by learners.

(Table 1 generated from: Anthony et al., 2015; Boaler, 2016; Boaler et al., 2017, 2018a, 2018b; Herbert, 2018; Hunter et al., 2018; Jorgensen & Dole, 2011; Sullivan et al., 2012; Sullivan & Lilburn, 2017)

Choosing a task that meets the criteria above does not automatically mean it will be rich when it is implemented in the classroom, however. Effective enactment is dependent on how the task is used by the teacher (Griffin, 2009).

IMPLICATIONS FOR TEACHERS

The level of teacher self-efficacy can contribute to teachers being hesitant to change the power balance in the mathematics classroom (Hunter, 2010; Zimmerman, 2000). Teachers' confidence with mathematical content is critical when developing inquiry-based mathematics communities in classrooms incorporating mixed ability grouping and appropriate task selection (Stipek et al., 2001). Current theory about how children best learn mathematics and the challenges this may present for teachers has implications for their mathematics planning.

Planning for guided inquiry

Traditionally, mathematics planning comprised a unit overview and a series of lesson plans based around a variety of activities that targeted different ability levels. Teachers commonly used commercially developed unit plans, including plans downloaded from the Internet. Using ready-made plans or templates saves time, but they do not compel teachers to actively and critically engage in the mathematics contained therein. In contrast, when teachers plan for guided inquiry and responsive teaching they need to be fully cognisant of the mathematical scope of tasks selected to guide learners with an appropriate level of challenge. The process described may help teachers to develop a community of learning in their classrooms through "a more responsive style [of teaching] that takes seriously views, beliefs and conceptions of the learner" (Fraser, 2016, p. 59). Student achievement gains are closely linked to teacher content knowledge in mathematics (Hill et al., 2005) further emphasising the importance of teacher engagement in mathematical content during the planning phase of teaching mathematics. Planning in guided inquiry seeks to reflect the responsive nature of teaching, while preparing teachers with specific mathematical content for explicit acts of teaching as they occur.

Enacting guided inquiry pedagogies, and confidently facilitating mixed ability grouping while also meeting the criteria of rich tasks, requires teachers to have a deep understanding of mathematical content. During planning, mathematical confidence can be developed through teacher collaboration when developing mathematical progressions and then reinforced when planning learning experiences for a specific group of learners based on these progressions. The planning process offered is broken down into two very distinct phases: planning the unit overview and planning for learning experiences. The overview component ensures the mathematics encompassing the unit is explicit. The learning experience component allows teachers to make specific decisions for their learners, while drawing on the mathematical progressions generated in the overview. Teachers are encouraged to plan unit overviews collaboratively as collaboration encourages teacher engagement in mathematics content as they discuss the progression of content relevant to their learners. Rather than offer a template, Table 2 (below) shows suggested components of the Unit Overview and a rationale. Schools can use these suggestions to develop a system of planning that meets the needs of teachers, learners and maths programmes.

A significant difference between traditional planning and planning for guided inquiry is the purpose of learning intentions. Traditionally, learning intentions are specific, often including a skill e.g. 'We are learning how to use rounding and compensating'. In guided inquiry, this becomes 'We are learning to solve problems using a variety of strategies'. Where 'rounding and compensating' may be one of the strategies, the over-arching concept is that children are learning to think mathematically about problems (Boaler, 2016).

As noted in the table above, the overview includes the progression of mathematical concepts and skills needed by learners to engage with the mathematical content. The overview also includes explicit prior knowledge, including mathematical understanding and skills, needed for the content being targeted. Teachers who trialled this process began by brainstorming mathematical content using resources they were already familiar with, such as www.nzmaths.co.nz and resources in the school such as Jo Boaler's *Mathematical Mindsets* (2016). It is noted that teachers who previously planned independently expressed a growing confidence in their own mathematical understanding when discussing the progression of the mathematics content with others. [2]

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[2] Currently unpublished data

Table 2.
Unit Overview

Components for consideration	Rationale/Explanation
Achievement Objectives	Taken directly from the New Zealand Curriculum (2007) document; choose AOs that are relevant to the learning.
Learning Intentions	Articulates the general concepts of maths for learning. These are decontextualised and can be applied to a variety of contexts.
Key Ideas	The break-down of the maths being experienced within the unit. That is, the mathematical concepts and content that learners might use, or be guided to use, during the learning experiences and may include a variety of strategies, and a variety of approaches which gives teachers a focus when choosing tasks.
Prior Knowledge	Encompasses skills, strategies, understanding and knowledge learners need to engage in the tasks that link to the mathematical content. Offers early indicators of scaffolding learners may need.
Success Criteria	What the teacher will see/ hear learners doing or saying if they meet the learning intentions. Success criteria are very closely linked to the key ideas and can be met at varying stages of the unit.
Curriculum Integration	Used to identify authentic connections with other curriculum areas to give mathematics a context and purpose.
Other: Ngā tikanga me te reo Māori Key Competencies Values	Anything for which there is a schoolwide focus that this unit will be used to explicitly reinforce.

Once the progression of mathematical concepts and content have been identified collaboratively, teachers can refer to the unit overview for explicit content and skills as they choose rich tasks as a context for mathematics learning experiences for their own group of learners.

Planning for learning experiences

A paradigm shift with guided inquiry is while teachers are actively engaged in children's learning, they may not know exactly how the lessons will proceed until they observe and notice how learners approach tasks or respond to questions. (Anthony et al., 2015). Instead of planning for directing what learners will do, teachers need to plan for how to scaffold or extend learners' thinking around a mathematical concept using a carefully selected task. Using the criteria for rich tasks in Table1, teachers can select, create and adapt tasks from a variety of sources when planning learning experiences. Accordingly, scaffolding and extending questions are included in planning for learning experiences. Table 3 (below) suggests components to consider when planning for learning experiences.

Table 3.
Planning for Learning Experiences

Components to consider	Explanation/Rationale
Learning Intentions	Include relevant intentions from overview
Rich Task	The task selected for learners to investigate – see Table 1: Criteria for a Rich Task.
Open Question	The question that will introduce the task and stimulate mathematical thinking.
The mathematical ideas	The maths relevant to the task can be cut and pasted from the key idea section of overview. These ideas are what the teacher is looking and listening for as the learners respond to the initial open questions and as learners engage in the task.
Scaffolding questions	These questions prepare the teacher to support learners to clarify their thinking or encourage a different way of thinking. They may take the form of "what if" or "how could you …." "What do you notice …." type of questions. (see "Using talk moves" http://teach.conceptualmath.com/talk-moves)
Extending questions	These are questions prepared for learners who demonstrate an understanding of the mathematical concepts identified for the task and will extend or consolidate learners' current thinking. Extending questions may add a new element to the task.
Workshops	These are responsive teaching sessions on an 'as needed' basis some will be planned in advance, others planned as the unit progresses.
Checkpoints	Learners articulate and share their thinking at various points of the task. This can be done as a group, between groups or as a class discussion. Checkpoints are linked to the success criteria in the unit overview and can expose learners to the mathematical thinking of others.

Using the components above, teachers can plan learning experiences around rich tasks that meet the needs of a diverse range of learners. Mathematical clarity for teachers is enhanced while preparing scaffolding and extending questions matching the key mathematical ideas identified in the overview. Creating scaffolding and extending questions linked to the relevant mathematical ideas enables teachers to critically engage in the mathematical progressions. The success of a planned learning experience relies on teachers noticing what learners are doing and saying, and responding in a way that encourages deeper mathematical thinking and understanding. (Anthony et al., 2015).

Including the relevant mathematical ideas with the prepared questions while planning learning experiences supports teachers to 'notice' mathematical thinking and ideas when learners are explaining and representing their thinking. Identifying required prior knowledge or skills in the overview can also help teachers to prepare for mathematical concepts that may need to be highlighted as learning experiences evolve. While mathematical content for explicit teaching may be apparent to the teacher as they are planning, workshops for learners are utilised when a need is identified within a task to allow opportunity for learners to be challenged first (Ingram et al., 2019). Further explicit teaching needs are likely to arise as the unit progresses, and are planned for in response to learner need. Teaching and planning in response to learners' needs epitomises the unknown component of guided inquiry and is again, reliant on teachers noticing and responding to learners' approaches, reasoning and thinking about a task. Ingram et al. (2019) identify the need for learners to attempt tasks before teacher intervention to enable learners to use and show their existing mathematical thinking and to increase ownership of their learning. Furthermore, a characteristic of planning for learning experiences in guided inquiry, is that a rich task at the base of an experience may evolve over several teaching sessions, depending on learners' mathematical understanding and response to the initial task.

Recognising the significance of learners' mathematical thinking and where it fits into the progression of mathematics learning is supported by the collaborative nature of identifying explicit key ideas in the first stage of planning for the overview. Teachers are further prepared for how learners think when creating scaffolding and extending questions linked to a specific task.

CONCLUSION

Recent changes in primary mathematics in the classroom have implications for decisions made when grouping and responding to learners and how teachers plan for their learners. This article suggests a planning approach teachers can adopt as support for making a pedagogical shift from teacher directed styles of teaching to a model of guided inquiry with an emphasis on learner agency and a student-centred maths environment. I argued at the beginning of this article that there are three aspects fundamental to teacher practice that increase student learning opportunities in primary mathematics: guided inquiry, grouping, and task selection. When using guided inquiry in mathematics, it is important that teachers fully understand the mathematical concepts to support them recognising the implications of learners' approaches to tasks. Grouping decisions will impact on the type of tasks teachers set and how they respond to learners' engagement with the task. As teachers notice how learners approach a task, including representations and language used, they can include explicit acts of teaching through carefully phrased questions or suggestions. Using the stages of this planning process requires teachers to critically engage in the purpose and content of the mathematics whilst collaboratively planning the unit overview. Further opportunities for teachers to critically engage in the mathematics of the unit, occur when selecting rich tasks that link to the progressions identified in the overview, and when preparing both scaffolding and extending questions. This means it is critical to select tasks which allow for a variety of approaches, responses and representations by learners. Critical engagement in the planning phase supports teachers to develop a mathematical inquiry community with learner agency at the forefront.

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