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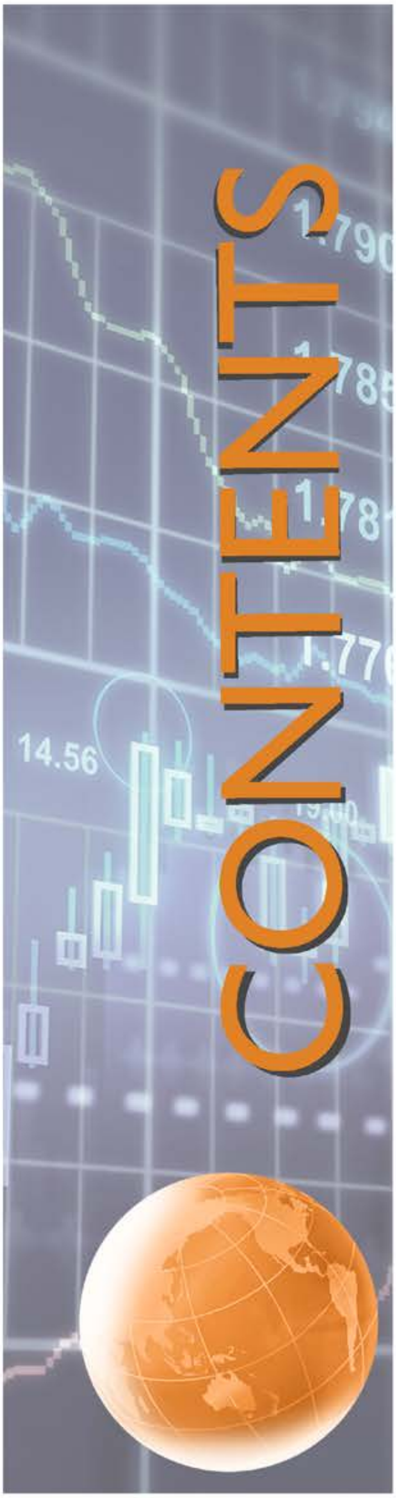

APPLIED FINANCE LETTERS

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THE PRICE TRANSMISSION IN EUROPEAN STOCK MARKETS

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Abstract: We investigate the dynamic price relationships among ten major stock indexes in Europe before, during and after the recent financial crisis. Using an error-correction model we find that the stock markets are cointegrated with three cointegrating vectors before the crisis and only one cointegrating vector during and after the crisis. We further use directed acyclic graph (DAG) analysis to explore the instantaneous transmission pattern. Contrary to previous research, the UK market is consistently mapped as being caused by several other markets, and France and Spain appear to share leadership roles before the crisis, while leadership is less evident during and post crisis. We also find a decreasing number of instantaneous casual relationships between the markets after the crisis, indicating that the markets are becoming more independent. This result is corroborated by a decline in the number of cointegrating vectors from pre to post crisis.

Keywords: error correction model (ECM), cointegration, directed acyclic graphs (DAG), financial crisis

1. Introduction

Several studies in the financial literature have investigated market linkages and price transmission mechanisms in the major international equity markets, employing the analytical framework of the vector auto-regression (VAR) or the error correction model (ECM). However, virtually all of these models rely on some form of temporal causality. Yang and Bessler (2004) extended the literature by using the method of directed acyclic graphs (DAGs) in combination with error correction modelling to explore evidence of contemporaneous causal patterns in international equity market data.

This paper extends this literature by adopting the techniques in Bessler and Yang (2004) to provide evidence of structural change in stock market linkages and price transmission in response to the 2007-2012 financial crisis. We first divide post-2000 weekly stock index data from ten prominent European markets into three periods representing pre-crisis (2000-2006), crisis (2007-2012), and post-crisis (2013-2016). We then evaluate price transmission between these markets during these three periods using an error-correction model to compute an innovation correlation matrix for each period and corresponding DAG and compare the results.

The paper is organized as follows. Section 2 provides data and summary statistics. Section 3 discusses methodology. Section 4 explains the empirical results of our error correction modelling and DAG analysis. Section 5 presents the conclusions.

2. Data

We use weekly time series for ten European equity indexes in local currency terms (Belgium: BEL 20, France: CAC 40, Germany: DAX, Greece: ASE, Ireland: ISEQ, Italy: FTSE MIB, Netherlands: AEX, Spain: IBEX 35, Switzerland: SMI, and the UK: FTSE 100), from January 2000 to May 2016 (857 observations for each index). All indices are rescaled to start at 100 at the beginning of the period. Figure 1 depicts weekly time series for the ten indices. All markets experienced a substantial run-up prior to 2007, and precipitous decline during financial crisis through 2012. Beginning 2012, all markets (with the exception of Greece and Italy), participated in a recovery, with the German index showing the strongest upward trend. Table 1 provides the corresponding summary statistics. Note that Ireland's market exhibits the greatest volatility, and Greece exhibits the worst performance (pre and post crisis).

Figure 1: Index Performance 2000-2016

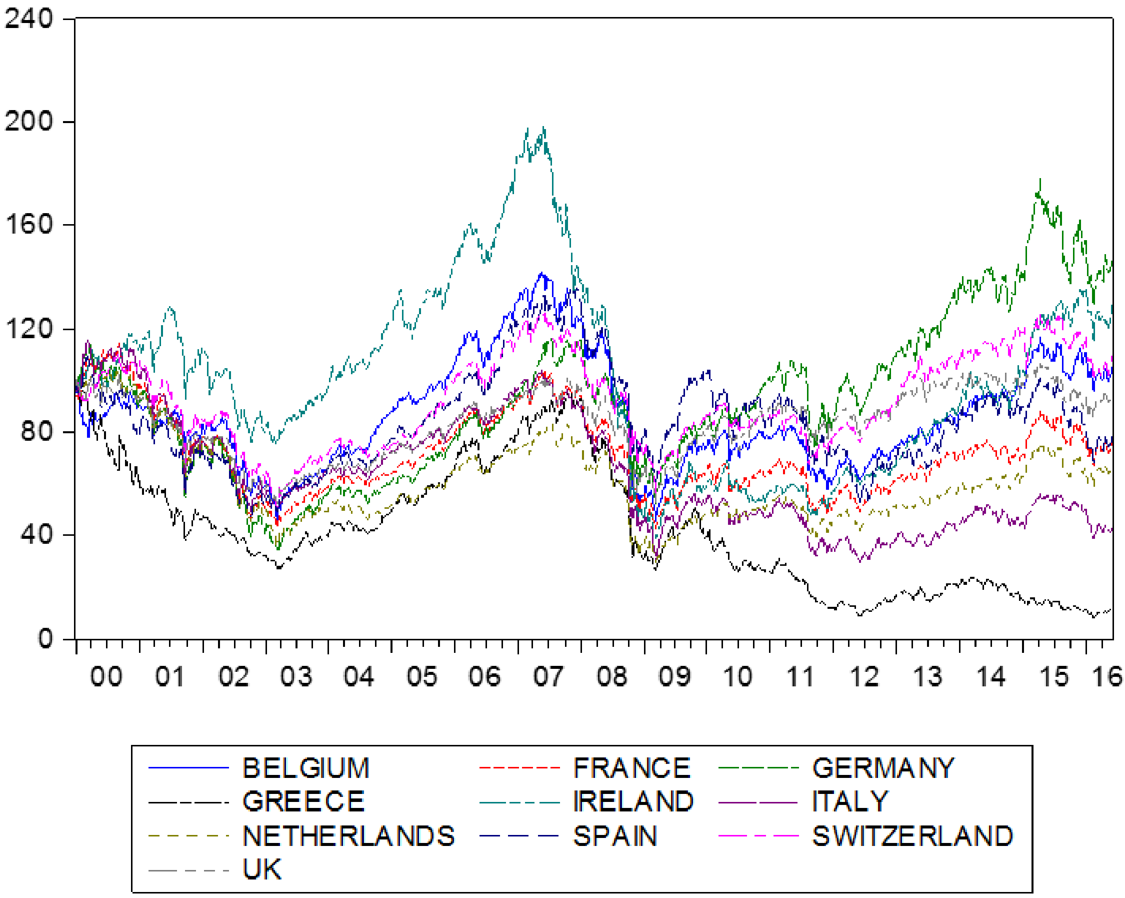


Table 1: **Summary statistics. All indexes are rescaled to start with 100 at the beginning of January 2000**

	BE	FR	DE	GR	IE	IT	NL	ES	CH	UK
Mean	86.8	71.9	92.7	41.8	100.9	63.8	60.9	84.6	92.1	83.9
Median	83.6	69.9	89.7	38.8	99.7	56.1	57.9	83.4	90.0	86.2
Max	142.2	114.4	177.8	100.0	198.6	115.8	103.5	135.9	125.9	106.4
Min	45.7	42.5	34.5	8.2	38.9	29.9	29.7	47.2	51.3	52.4
Stdev	20.6	15.6	30.4	24.0	35.5	21.7	15.8	18.8	17.4	12.9
Skew	0.6	0.6	0.5	0.5	0.5	0.6	0.8	0.6	0.0	-0.5
Kurt	2.8	2.6	2.7	2.2	2.8	2.2	3.2	3.0	2.0	2.2
#Obs.	857	857	857	857	857	857	857	857	857	857

Source: finance.Yahoo.com

3. Methodology

3.1 Error Correction Modelling

Following Yang and Bessler (2004) and Refalo (2009), we first apply a cointegrated VAR model to evaluate the data. Letting X_t denote a vector of ten indexes (k=10), the corresponding vector ECM is specified as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu + \varepsilon_t \quad (t = 1, \dots, T) \tag{1}$$

$$\varepsilon_t \sim iid(0, \Sigma) \tag{2}$$

μ is a (k by 1) vector of intercepts, εt is the corresponding vector of white noise disturbance terms, and Γi are (k by k) coefficient matrices defining the short-run adjustments to changes in the price process. Of interest is evidence of a price transmission mechanism contributing to deviations in long-run relationships between market indexes. If the indexes are cointegrated, Π can be factored into two matrices, Π=$\alpha\beta$', where β is the cointegrating vector and α indicates the speed of adjustment to the previous period's deviation from the cointegrating relationship. The rank of Π determines the number of cointegrating vectors.

We apply Trace tests developed by Johansen (1991) to determine the number of cointegrating vectors. The test statistics is computed as:

$$Trace = -\sum_{i=r+1}^{k} T ln(1 - \lambda_i^*) \tag{3}$$

where λ^*_i are the estimated eigenvalue(s), T is the number of observations, and r is the maximum cointegrating rank. Rejection of the hypothesis implies the number of cointegrating vectors exceeds r. In order for the cointegration test to be valid, unit root tests are conducted on each series to test for non-stationarity before we apply the cointegration test.

3.2 Directed Acyclic Graphs

The method of directed acyclic graphs (DAG) uses a series of logic based rules to deduce contemporaneous causal relations from the correlation structure of a dataset. It is applied by first determining which variables are un-conditionally or conditionally correlated, and then by using a series of logic arguments (known as sepset conditions) to determine causal direction of these correlations, creating a causal map linking the variables. The advantage of this method is that it requires no ad-hoc or theoretical restrictions (though such restrictions may be employed) in determining links or causality. This paper uses TETRAD 5.2.1 software for constructing the DAGs.

In our application, one begins with a diagram of the ten markets connected to each other by straight lines (links), each representing the correlation between those markets. Links between markets that are not statistically correlated are eliminated. The remaining links are then turned into arrows (using the sepset conditions in a stage known as orientation) indicating the causal direction of correlation. The resulting graph (or DAG) indicates the pattern of contemporaneous causality between the ten markets. Note that in this paper, we apply no exogenous or structural restrictions in determining our DAGs, and eliminate all links that are not significant at the .01 level.

The method is extensively discussed in science literature in Spirtes et al (2000) and Glymour and Cooper (1999). DAG has been applied to studying financial data in a number of other papers including Bessler and Yang (2003), Yang and Bessler (2004), Haigh et al (2004), and Li et al (2008), and Refalo (2009). The latter two papers provide a detailed overview of the algorithm (Li et al 2008 illustrate how the algorithm works graphically). More recently Jayech (2011) studies the August 2011 stock market crash with a DAG-copula based approach using daily returns of stock indices and bonds.

4. Empirical analysis

4.1 Error Correction Modelling and Cointegration Tests

Table 2 presents results from Johansen cointegration tests. The tests are conducted without a drift term in the VAR; all tests assume a constant in the cointegrating vector(s). Testing is ended at the first failure to reject the hypothesis; the Akaike information criterion was used to select the number of VAR lags used (one for pre-crisis period and two for crisis and post-crisis periods). The results are three cointegrating vectors linking the markets in the pre-crisis period, and only one cointegrating vector linking the markets crisis and post-crisis, indicating reduced market cointegration after the crisis began. The cointegrating ranks we observe are consistent with Bessler and Yang (2003) which notes that stock price series tend to exhibit fewer cointegrating vectors, indicating a loose long-run co-movement among stock market prices. Likewise, international equity market studies using cash indices by Francis and Leachman

(1998) and Masih and Masih (2001) find only one (or no) cointegrating vector linking the markets.[1]

Table 2: Johansen's Cointegration Tests

Reported are the Trace test statistics, under a hypothesis H0 of zero to three cointegrating vectors. Where T is the number of observations, r is the maximum number of cointegrating vectors, n is the number of eigenvalues, and λi^* is the estimated eigenvalue, the statistics are given by Equation (3). Results displayed are for the three sub-sample periods 2000-2006, 2007-2012 and 2013-2016.[2]

	2000-2006			2007-2012			2013-2016		
	VAR lag=1			VAR lag = 2			VAR lag = 2		
H0 rank	**Trace**	**C(5%)**	**Decision**	**Trace**	**C(5%)**	**Decision**	**Trace**	**C(5%)**	**Decision**
None	345.6	251.3	R	253.9	251.3	R	263.1	251.3	R
At most 1	249.4	208.4	R	190.7	208.4	F	181.5	208.4	F
At most 2	177.8	169.6	R	150.6	169.6	F	131.8	169.6	F
At most 3	132.4	134.7	F	113.2	134.7	F	98.3	134.7	F

4.2 Error Correction Modelling and Cointegration Tests

The ECM yields the innovation correlation matrices (4.1-4.4), with the markets listed in the order Belgium, France, Germany, Greece, Ireland, Italy, Netherlands, Spain, Switzerland, and UK, for pre-crisis, crisis, post-crisis, and all periods combined:

$$\Sigma_{precrisis} = \begin{bmatrix} 1 \\ 0.66 & 1 \\ 0.67 & 0.89 & 1 \\ 0.32 & 0.41 & 0.39 & 1 \\ 0.58 & 0.58 & 0.60 & 0.25 & 1 \\ 0.66 & 0.87 & 0.85 & 0.38 & 0.57 & 1 \\ 0.76 & 0.89 & 0.87 & 0.40 & 0.60 & 0.86 & 1 \\ 0.63 & 0.80 & 0.79 & 0.38 & 0.53 & 0.78 & 0.77 & 1 \\ 0.76 & 0.72 & 0.71 & 0.30 & 0.56 & 0.72 & 0.79 & .64 & 1 \\ 0.68 & 0.84 & 0.81 & 0.31 & 0.57 & 0.79 & 0.83 & 0.72 & 0.76 & 1 \end{bmatrix} \tag{4.1}$$

$$\Sigma_{crisis} = \begin{bmatrix} 1 \\ 0.90 & 1 \\ 0.82 & 0.92 & 1 \\ 0.70 & 0.67 & 0.60 & 1 \\ 0.74 & 0.72 & 0.65 & 0.59 & 1 \\ 0.87 & 0.93 & 0.84 & 0.67 & 0.67 & 1 \\ 0.90 & 0.93 & 0.87 & 0.68 & 0.72 & 0.86 & 1 \\ 0.79 & 0.87 & 0.81 & 0.63 & 0.59 & 0.86 & 0.78 & 1 \\ 0.83 & 0.87 & 0.81 & 0.60 & 0.70 & 0.83 & 0.85 & 0.75 & 1 \\ 0.86 & 0.93 & 0.89 & 0.63 & 0.70 & 0.86 & 0.91 & 0.79 & 0.86 & 1 \end{bmatrix} \tag{4.2}$$

[1] We also test whether each price series is itself stationary and conduct additional tests for the restrictions on the cointegration space.

[2] Johansen's cointegration tests are also performed with the whole sample 2000-2016. Trace test indicates two cointegrating equations at the 0.05 level with two VAR lags.

$$\Sigma_{postcrisis} = \begin{bmatrix} 1 \\ 0.92 & 1 \\ 0.89 & 0.92 & 1 \\ 0.39 & 0.41 & 0.37 & 1 \\ 0.73 & 0.74 & 0.72 & 0.38 & 1 \\ 0.80 & 0.85 & 0.79 & 0.56 & 0.66 & 1 \\ 0.89 & 0.94 & 0.90 & 0.39 & 0.73 & 0.82 & 1 \\ 0.80 & 0.86 & 0.78 & 0.54 & 0.60 & 0.88 & 0.82 & 1 \\ 0.64 & 0.62 & 0.61 & 0.25 & 0.51 & 0.50 & 0.62 & 0.56 & 1 \\ 0.76 & 0.82 & 0.77 & 0.34 & 0.58 & 0.70 & 0.84 & 0.70 & 0.65 & 1 \end{bmatrix} \quad (4.3)$$

$$\Sigma_{all} = \begin{bmatrix} 1 \\ 0.81 & 1 \\ 0.77 & 0.89 & 1 \\ 0.50 & 0.51 & 0.43 & 1 \\ 0.68 & 0.66 & 0.61 & 0.43 & 1 \\ 0.76 & 0.89 & 0.78 & 0.50 & 0.61 & 1 \\ 0.83 & 0.91 & 0.83 & 0.49 & 0.65 & 0.85 & 1 \\ 0.75 & 0.83 & 0.78 & 0.51 & 0.59 & 0.82 & 0.76 & 1 \\ 0.76 & 0.76 & 0.72 & 0.42 & 0.60 & 0.71 & 0.76 & 0.68 & 1 \\ 0.79 & 0.86 & 0.81 & 0.45 & 0.62 & 0.78 & 0.84 & 0.75 & 0.78 & 1 \end{bmatrix} \quad (4.4)$$

Unconditionally the correlations between countries are similar in magnitude to the results of Yang and Bessler (2004), which uses country stock future index data. This result is unsurprising given the degree of economic integration among markets in this study.

Dividing the data into pre-crisis, crisis, and post-crisis periods, we find that the instantaneous correlations are greater during the crisis period. To test the significance of this change in correlation between periods, we employ the Z-test with the Fisher transformation, Fisher (1921). First, we transform each correlation coefficient using Equation (5):

$$\rho' = 0.5 \ln(\frac{1+\rho}{1-\rho}) \quad (5)$$

We then test for statistical significance in the difference in correlation for each element of the innovation correlation matrix between any two periods by computing the z-statistic and corresponding p-value:

$$z = \frac{\rho_1' - \rho_2'}{\sqrt{\frac{1}{N_1-3} + \frac{1}{N_2-3}}} \quad (6)$$

Matrices 7.1-7.3 display changes in the correlation coefficients from pre-crisis to crisis, crisis to post-crisis, and pre-crisis to post-crisis, which are significant at the 5% significance level. A 1 for an increase, -1 for a decrease, and 0 for no statistically significant change:

$$\Sigma_{crisis} - \Sigma_{precrisis} = \begin{bmatrix} NA & & & & & & & & & \\ 1 & NA & & & & & & & & \\ 1 & 1 & NA & & & & & & & \\ 1 & 1 & 1 & NA & & & & & & \\ 1 & 1 & 0 & 1 & NA & & & & & \\ 1 & 1 & 0 & 1 & 1 & NA & & & & \\ 1 & 1 & 0 & 1 & 1 & 0 & NA & & & \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & NA & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & NA & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & NA \end{bmatrix} \quad (7.1)$$

$$\Sigma_{postcrisis} - \Sigma_{crisis} = \begin{bmatrix} NA & & & & & & & & & \\ 0 & NA & & & & & & & & \\ 1 & 0 & NA & & & & & & & \\ -1 & -1 & -1 & NA & & & & & & \\ 0 & 0 & 0 & -1 & NA & & & & & \\ -1 & -1 & 0 & 0 & 0 & NA & & & & \\ 0 & 0 & 0 & -1 & 0 & 0 & NA & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & NA & & \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & NA & \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & NA \end{bmatrix} \quad (7.2)$$

$$\Sigma_{postcrisis} - \Sigma_{precrisis} = \begin{bmatrix} NA & & & & & & & & & \\ 1 & NA & & & & & & & & \\ 1 & 0 & NA & & & & & & & \\ 0 & 0 & 0 & NA & & & & & & \\ 1 & 1 & 1 & 0 & NA & & & & & \\ 1 & 0 & -1 & 1 & 0 & NA & & & & \\ 1 & 1 & 0 & 0 & 1 & 0 & NA & & & \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & NA & & \\ -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & NA & \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & NA \end{bmatrix} \quad (7.3)$$

Compared with pre-crisis levels, 38 out of 45 correlation coefficients are statistically greater during the crisis. The majority of the correlation coefficients then decrease following the crisis. Comparing pre-crisis and post crisis pairwise correlations, the Belgian, Irish, and Greek markets generally exhibit greater correlation with the other markets post-crisis, and the Swiss and UK markets show evidence of reduced post-crisis pairwise correlation.

4.3 DAG Analysis

To study evidence of instantaneous casualty and structural changes in the pattern of causality for pre-crisis, crisis, and post-crisis periods, we construct DAGs for the innovation correlation matrices 4.1-4.4, respectively. As discussed in section 3.2, all graphs are estimated requiring a .01 significance level for correlation between markets.
Figure 2A presents the DAG for the pre-crisis period. There are ten directional links (including one bidirectional link) and five non-directional links. The graph indicates a direct causal flow from France, Germany, and Switzerland to the UK, with Netherlands and Spain causing UK indirectly via Germany. This result differs from Yang and Bessler (2004), which finds the UK to have a leadership role among the European markets. The difference in findings may be due to the different data span (1997-2007 vs 2000-2006), frequency of the data (daily vs weekly), instrument (index future vs index), and the number of European countries examined in our studies (four versus ten). France and Spain are graphed as having leadership roles. Our DAG also reveals changes in the Italian market to be driven by trading in several other markets. There are links between Switzerland, Netherlands, Belgium, and Ireland, but in general, there is little evidence of a directional causality pattern among those markets.

That Germany is mapped as being caused by several lesser markets may be explained by the one to three hour delay in closing times of the German Exchanges (19:00 and 21:00 UTC) vis-a-vis the other European exchanges, allowing for additional trading in the German market in response to last minute trades in markets that have closed. Our results are consistent with an integrated market prior to the financial crisis – with many market indices moving simultaneously in response to contemporaneous information, and the difference in market closing times explaining why the German index is graphed as a follower. Also note that Greece, the nation which will later face a sovereign debt crisis in 2009, is shown as an outlier in the pre-crisis period, not having any casual flow to or from other countries.

Figure 2B presents the DAG for the crisis period 2007-2012 and has a different structure. Most countries have casual flows to and/or from other countries. The UK market is influenced directly by Germany, Netherlands, France, and Switzerland. Ireland and Greek markets are now part of the causal diagram as being influenced directly or indirectly by trading in virtually all other markets. France has direct links with six markets, though only one causal relationship is mapped – France causing UK. During this period, the German and Netherlands markets are graphed as having leadership roles. The diagram is consistent with centralized government (EU) policy changes and trading in quality markets driving the markets that are in crisis. It is also consistent with investor flight to quality, where the markets in greatest crisis become followers.

Figure 2C presents the DAG for post-crisis period 2013-2016. Only two directional links and seven non-directional links are found. Again, UK is graphed as being caused by the other markets, and France has the greatest number of direct relationships. The Ireland index is graphed as having no causal flow with the other European markets, possibly reflecting that continued domestic policy turmoil has the greatest influence on trading in that market. The reduction in linkages from pre-crisis may indicate a greater degree of independence among these markets and could be the result of reforms implemented after the crisis to reduce risk taking and financial contagion. However, the presence of mostly non-directional links indicates simultaneous price movements and market integration.

Comparing the DAGs for the three periods, we find evidence consistent with centralized government policy making and investor flight to quality influencing the pattern of price information transmission during the crisis, and greater market independence ex-post the crisis, using a contemporaneous time analysis. This is consistent with VAR model analysis in which the number of cointegrating vectors in the data declined from three vectors pre-crisis to one post-crisis.

Figure 4D presents a DAG analysis for the entire sample (2000-2016). While there are a number of bidirectional relationships, what stands out is that the UK market is graphed as being caused by the other key markets. This is consistent with the results of our pre-crisis, crisis, and post-crisis graphs.

Figure 2A: Pre-Crisis Pattern from TETRAD V

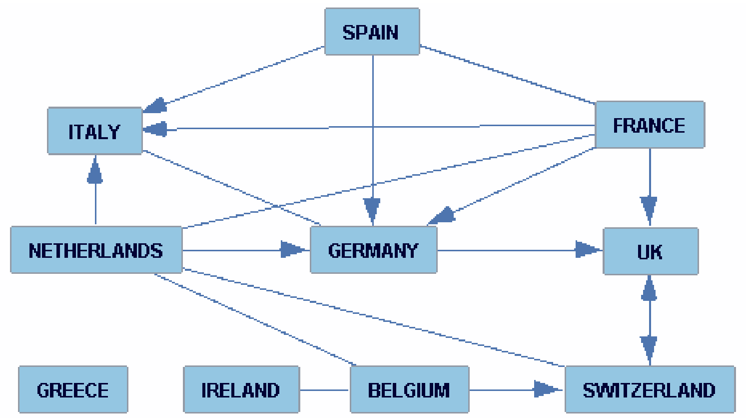


Figure 2B: Crisis Pattern from TETRAD V

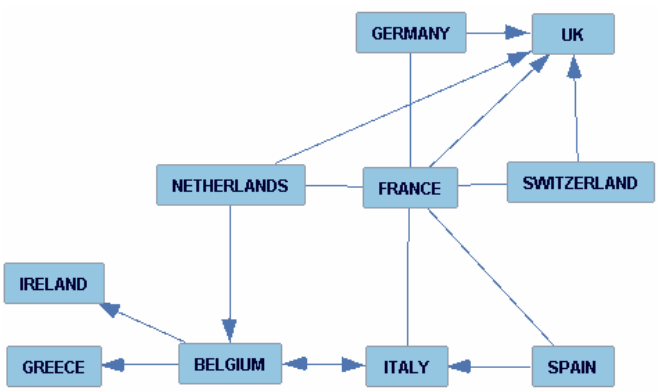


Figure 2C: Post-Crisis Pattern from TETRAD V

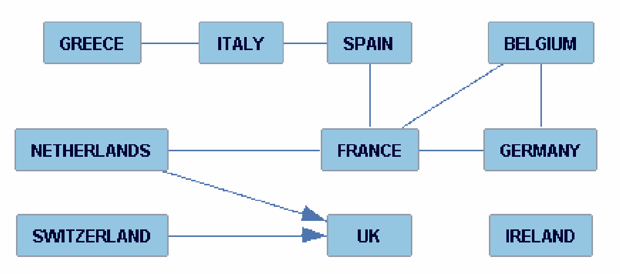
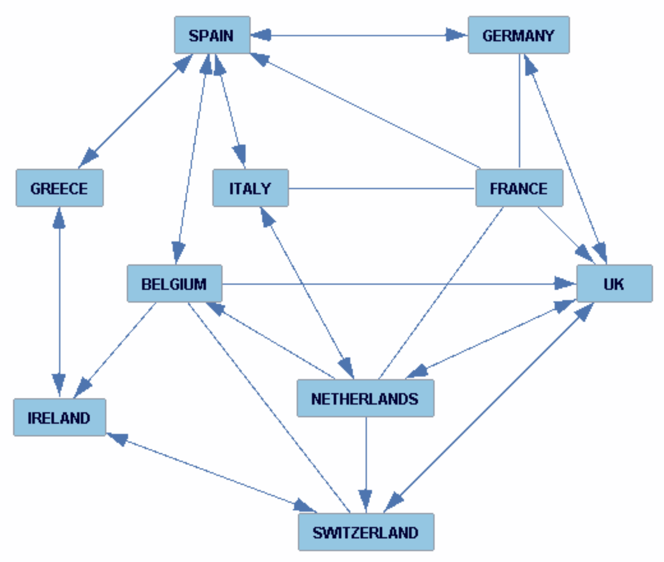


Figure 2D: Whole Sample Pattern from TETRAD V



5. Conclusion

We investigate price transmission patterns in the ten European stock indexes before, during, and after the Great Recession following the approach of Yang and Bessler (2004), which combines cointegration, ECM, and DAG methodologies. Different from recent studies such as Francis and Leachman (1998), Masih and Masih (2001), and Bessler and Yang (2003), where only one cointegrating vector is found among major stock markets, and from Yang and Bessler (2004) where two cointegrating vectors are found, our ECM analysis indicates that there are three cointegrating before the crisis and only one cointegrating vector in the other periods.

We then study instantaneous causality between these markets using DAGs. France and Spain appear to share leadership roles before the crisis while Germany and Netherlands become leaders during the crisis. Contrary to previous research, the UK is consistently graphed as being caused by other markets (though this becomes more pronounced during the crisis), and the Irish and Greek market indices are graphed as being caused by other market indices during the crisis period. We also find a decrease in the number of instantaneous casual links between the markets after the crisis, with most links becoming non-directional, indicating greater independence of the European markets. This result is consistent with the results of our VAR model, and may be a result of post-crisis regulatory reforms to reduce risk taking and potential financial contagion in response to the stock market meltdowns. The impact of reform mechanisms on the European market linkages (and trading) is a sweeping topic that deserves extensive research but is beyond the scope of this paper.

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CAUSALITY BETWEEN STOCK MARKET AND "FEAR GAUGE" INDICES: AN EMPIRICAL ANALYSIS WITH E-STATISTICS

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Abstract: This study investigates empirically the validity of three hypotheses that have been advanced to explain the tendency of stock market and volatility indices to move in opposite directions, using the notion of Brownian distance correlation. We consider three stock market-implied volatility index pairs, namely, the S&P 500 and the VIX, the DAX 100 and the V1XI, and the N225 and the JNIV. The empirical results support the leverage hypothesis relative to the volatility feedback hypothesis for the pairs S&P 500 and VIX, and N225 and JNIV, and the representativeness and affect heuristics hypothesis relative to the leverage hypothesis for the pairs DAX 100 and V1XI, and N225 and JNIV.

Keywords: Brownian distance, stock index, volatility index

1. Introduction

The negative correlation between stock market and volatility returns has been well documented in Finance literature suggesting a potential diversification benefit to including volatility in an investment portfolio (e.g. Badshah, 2013; Bollerslev et al., 2006; Whaley, 1993; Campbell and Hentschel, 1992; Black, 1976). At the same time, however, there is a little agreement among researchers concerning the mechanism behind the tendency of stock market indices and volatility indices to move in opposite directions. Leading explanations include the leverage hypothesis (Christie, 1982; Black, 1976), the volatility feedback or time-varying risk premium hypothesis (Campbell and Hentschel, 1992; French et al., 1987), and the representativeness and affect heuristics hypothesis (Badshah, 2013; Hibbert et al., 2008). The first attributes the negative relationship between stock market volatility returns to the financial leverage of firms (i.e. stock price declines render firms with a high debt-to-equity ratio riskier). The second suggests that a rise in expected volatility causes current stock prices to drop so that investors can be compensated for the extra risk involved. The third focuses on stereotypes and rules of thumb or short-cuts used by people to make judgements when are busy or under time pressure (for example, they expect higher returns with lower risk from stocks of financially stable firms or they link, without any high-level reasoning, benefits with something "positive" and risks with something "negative")[1].

[1] For further details see Badshah (2013), Shefrin (2008), and Finucane et al. (2000).

From an empirical perspective, the fundamental difference between the three competing hypotheses lies in their respective implications about causality. The leverage hypothesis implies that changes in stock returns lead changes in volatility; the time-varying risk premium hypothesis implies exactly the opposite causal order; the representativeness and affect heuristics hypothesis predicts a contemporaneous than a lead-lag relationship between stock market and volatility indices.

The presence and the direction of causality between the two variables is important for investors aiming to profit from the stock and the volatility derivatives markets (Chiang, 2012). Earlier empirical investigations on the topic relied on a variety of approaches ranging from simple correlation and regression models to multivariate GARCH ones (e.g. Chiang, 2012; Hibbert et al., 2008; Bollerslev et al., 2006; Giot, 2005). Their results have been often conflicting depending on the time period considered, the statistic of volatility employed (realized or implied), and the analytical tools adopted.

This work revisits the contemporaneous and the lead-lag relations between stock market and volatility indices using notions and tools from Energy Statistics (E-statistics) (Szekely et al., 2007). Through them one may obtain a scale-invariant measure of general (linear and non linear) co-movement which, as shown by Creamer and Creamer (2016), may provide richer insights about the linkages among stochastic processes relative to alternatives. In what follows section 2 presents the analytical framework and section 3 the data, the empirical models and the results. Section 4 offers conclusions.

2. Analytical Framework

Let $X_i \ (i=1,2)$ be two random processes with characteristic functions f_i and joint characteristic function f_{12}. The *distance covariance*, $v(X_1, X_2)$, is the square root of $v^2(X_1,X_2) = \| f_{12}(s,t) - f_1(s) f_2(t) \|^2$ (where $\| \ \|$ is the norm and s and t are vectors) and measures the Brownian distance between f_{12} and $f_1 f_2$. Likewise, the *distance variance*, $v(X_i)$, is the square root of $v^2(X_i) = \| f_{ii}(s,t) - f_i(s) f_i(t) \|^2$. Once the distance covariance and variance are defined, the *Brownian distance correlation* $R(X_1, X_2)$ can be derived as

$$(1) \quad R^2(X_1,X_2) = \begin{cases} \dfrac{v^2(X_1,X_2)}{\sqrt{v^2(X_1)v^2(X_2)}}, & v^2(X_1)v^2(X_2) > 0 \\[2ex] 0, & v^2(X_1)v^2(X_2) = 0 \end{cases}$$

(Szekely and Rizzo 2013; Szekely *et al.*, 2007).

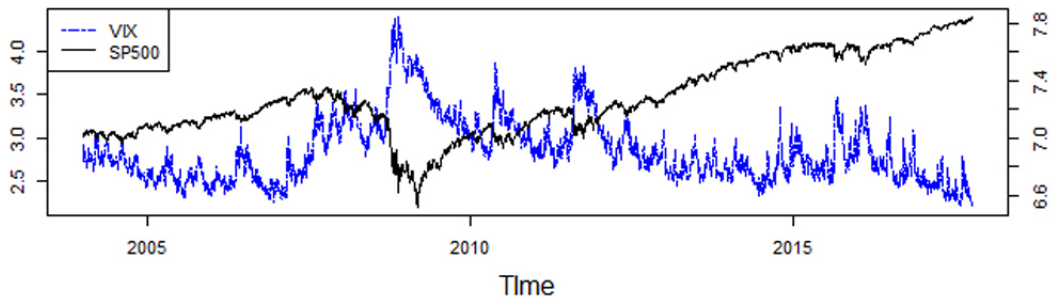
From the very definition of the norm, $v(X_1, X_2) \geq 0$ and $v(X_1, X_2) = 0$ *iff* the random processes X_1 and X_2 are independent. R is an unsigned correlation coefficient taking the value of 0 under independence and the value of 1 under perfect co-movement.

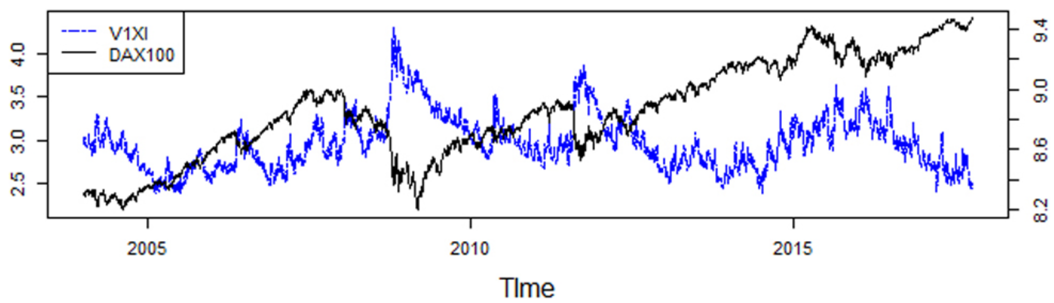
Provided that X_1 and X_2 consist of time series observations, the Brownian distance correlation may be used to investigate general co-movement of the current value of

X_i (X_{it}) on the l-lagged value of X_j (X_{jt-l}) $(j=1,2$ and $j \neq i)$. In particular, if $R(X_{it}, X_{jt-l}) > 0$ and l>0, then X_{jt-l} leads X_{it}. In addition, if $R(X_{it}, X_{jt-l}) > 0$ and $R(X_{it-l}, X_{jt}) = 0$, then there is a uni-directional relationship from X_{jt-l} to X_{it}. However, if $R(X_{it}, X_{jt-l}) > 0$ and $R(X_{it-l}, X_{jt}) > 0$, there is a feedback relationship between the two processes. In contrast, if $R(X_{it}, X_{jt-l}) = 0$ and $R(X_{it-l}, X_{jt}) = 0$, there is no lead-lag relationship between X_1 and X_2 (Creamer and Creamer, 2016).

3. The Data, the Empirical Models, and the Results.

The data for the empirical analysis are daily observations from three pairs of stock market and implied volatility (“fear gauge”) indices, namely, the S&P 500 and the VIX, the DAX 100 and the V1XI, and the N225 and the JNIV. They have obtained from the CBOE and the investing.com websites and they refer to the period 2/1/2004 to 6/10/2017 (a total of 3593 observations). As known “fear gauge” indices, are derived from stock options and represent a consensus forecast over the expected short-run (typically 30 calendar days) stock market volatility (e.g. Chiang, 2012; Whaley, 1993). Figure 1 presents the natural logarithms of the six time series. It is evident that, on most occasions, the stock and the implied volatility indices for a given market move in opposite directions.





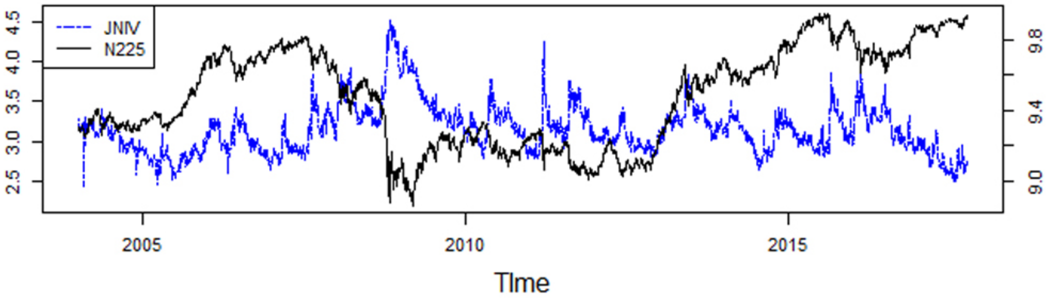


Figure 1. Logarithmic stock and implied volatility indices

Earlier empirical works (e.g. Giot, 2005) suggested that the strength and the pattern of the relationship between the stock market and the volatility indices may depend on volatility levels. Here, to allow for such possibility we have applied the multiple breakpoint test of Bai and Perron (2003) to the three log "fear gauge" indices and we have estimated the Brownian distance correlation coefficients at a number of different sub-periods. Table 1 (panels (a) to (c)) presents the test results. In all cases, the test detected four break points. It is noteworthy that the two first breaks occurred at about the same time for all log implied volatility series while the third and the fourth occurred at dates up to eight months apart. Also, the time periods between the first and the third break (which include the financial crisis of 2008/9 and the nervous years that followed) are characterized by higher implied volatility relative to the rest.

Table 1: Results of the Bai-Perron on the Log Implied Volatility Series

(a) VIX**

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	1313.567*	8.58	2/7/2007	2.6*
1 vs 2	2350.181*	10.13	7/9/2009	3.4*
2 vs 3	198.446*	11.14	26/1/2012	3.13*
3 vs 4	72.419*	11.83	21/1/2014	2.75*
4 vs 5	0	11.25		2.66*

**, Maximum no breaks: 5; trimming: 0.15; level of significance: 0.05(assessed using HAC standard errors)

(b) V1XI **

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	1065.662*	8.58	18/7/2007	2.76*
1 vs 2	720.402*	10.13	9/11/2009	3.21*
2 vs 3	582.111*	11.14	6/9/2012	3.01*
3 vs 4	127.117*	11.83	29/9/2014	2.73*
4 vs 5	0	12.25		3*

**, Maximum no of breaks: 5; trimming: 0.15; level of significance: 0.05 (assessed using HAC standard errors)

(c) JNIV **

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	898.871*	8.58	10/8/2007	2.96*
1 vs 2	817.738*	10.13	21/8/2009	3.57*
2 vs 3	182.723*	11.14	2/12/2011	3.22*
3 vs 4	24.075*	11.83	1/5/2014	3.16*
4 vs 5	0	12.25		3.05*

**, Maximum no of breaks: 5; trimming: 0.15; level of significance: 0.05 (assessed using HAC standard errors)

Prior to the estimations we have evaluated the stationarity of all time series using the ADF test. The log stock indices turned out to be non stationary for the total period and for all sub-periods. The log implied volatility indices turned out to be stationary in a number of sub-periods. All first log differences (returns), however, are stationary. To avoid mixing non stationary and stationary time series we have conducted the empirical analysis on returns.

Following Creamer and Creamer (2016), we have estimated Brownian distance correlations at l=1,2, …, 7 lags. Table 2 (panels (a) to (c)) presents the results . Starting with the pair (S&P 500, VIX), the Brownian distance correlations between current implied volatility returns and stock market returns at the different lags are all statistically significant for the total period and for the fifth sub-period; there is also a large number of statistically significant correlations in the remaining sub-periods, especially at $1 \leq l \leq 4$. The Brownian distance correlations between current stock market returns and implied volatility returns at the different lags are all statistically significant for the total period; there is also a relatively small number of statistically significant correlations in the remaining sub-periods, primarily at l=1. On the basis of the values and the statistical significance of the estimated distance correlations one may conclude that, although causality between the S&P 500 and the VIX may be bi-directional, the influence of lagged S&P 500 returns on current VIX returns has been far more stronger than that of lagged VIX returns on current S&P 500 returns. Therefore, between the leverage and the volatility feedback hypothesis the data appear to provide more support to the former.

Table 2: Brownian Distance Correlations

(a) S&P 500 and VIX returns

	Number of lags							
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
			Null hypothesis: SP500 does not lead VIX					
Total	0.207**	0.481**	0.086**	0.105**	0.076**	0.055**	0.059**	0.064**
Sub-period 1[+]	0.103**	0.561**	0.074	0.155**	0.094*	0.075	0.059	0.083
Sub-period 2	0.151**	0.597**	0.131**	0.102	0.111*	0.098	0.077	0.093
Sub-period 3	0.304**	0.434**	0.115*	0.133**	0.136**	0.070	0.099	0.076
Sub-period 4	0.296**	0.354**	0.085	0.109	0.106	0.081	0.097	0.077
Sub-period 5	0.282**	0.483**	0.115**	0.116**	0.094**	0.094*	0.092*	0.089*
			Null hypothesis: VIX does not lead SP500					
Total	0.207**	0.066**	0.067**	0.062**	0.062**	0.050*	0.051**	0.062***
Sub-period 1	0.103**	0.074	0.067	0.064	0.061	0.067	0.057	0.077
Sub-period 2	0.151**	0.124*	0.091	0.092	0.091	0.077	0.093	0.103
Sub-period 3	0.304**	0.102*	0.095	0.088	0.092	0.094	0.095	0.077
Sub-period 4	0.296**	0.113*	0.084	0.076	0.070	0.076	0.076	0.092
Sub-period 5	0.282**	0.103**	0.097*	0.097**	0.110**	0.080	0.065	0.086*

[+], the sub-periods 1 to 5 are: 3/1/2004 to 2/7/2007, 3/7/2007 to 7/9/2009, 8/9/2009 to 26/1/2012, 27/1/2012 to 21/1/2014, and 22/1/2014 to 6/10/2017, respectively; *, p≤0.05, **, p≤0.01

(b) **DAX 100 and V1XI**

	Number of lags							
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
			Null hypothesis: DAX 100 does not lead V1XI					
Total	0.772**	0.077**	0.083**	0.076**	0.072**	0.071**	0.048*	0.044*
Sub-period 1+	0.785**	0.132**	0.108**	0.066	0.091*	0.077	0.056	0.047
Sub-period 2	0.773**	0.109*	0.091	0.120*	0.104	0.113*	0.072	0.112*
Sub-period 3	0.818**	0.092	0.131**	0.117**	0.111**	0.110**	0.120**	0.067
Sub-period 4	0.799**	0.105	0.074	0.088	0.137**	0.087	0.091	0.097
Sub-period 5	0.725**	0.096*	0.105**	0.092*	0.080	0.075	0.066	0.063
			Null hypothesis: V1XI does not lead DAX 100					
Total	0.772**	0.056**	0.070**	0.055**	0.047*	0.055**	0.047*	0.043*
Sub-period 1	0.785**	0.057	0.077	0.080	0.048	0.085	0.080	0.060
Sub-period 2	0.773**	0.073	0.107*	0.085	0.101	0.079	0.113*	0.088
Sub-period 3	0.818**	0.110**	0.119**	0.099*	0.084	0.065	0.072	0.083
Sub-period 4	0.799**	0.096	0.085	0.074	0.083	0.072	0.102	0.086
Sub-period 5	0.725**	0.074	0.097*	0.078	0.094*	0.093*	0.055	0.065

+, the sub-periods 1 to 5 are: 3/1/2004 to 18/7/2007, 19/7/2007 to 9/11/2009, 10/11/2009 to 6/9/2012, 7/9/2012 to 29/9/2014, and 30/9/2014 to 6/10/2017, respectively; *, p≤0.05, **, p≤0.01

(c) **N225 and JNIV**

	Number of lags							
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
			Null hypothesis: N225 does not lead NIV					
Total	0.554**	0.113**	0.082**	0.077**	0.060**	0.061**	0.052**	0.055**
Sub-period 1+	0.420**	0.116**	0.092*	0.098**	0.071	0.088*	0.062	0.074
Sub-period 2	0.706**	0.137**	0.080	0.086	0.109	0.081	0.091	0.086
Sub-period 3	0.719**	0.124**	0.139**	0.113*	0.093	0.125*	0.081	0.119*
Sub-period 4	0.346**	0.128**	0.080	0.084	0.077	0.090	0.072	0.073
Sub-period 5	0.589**	0.125**	0.117**	0.107**	0.080	0.105**	0.083	0.092*
			Null hypothesis: JNIV does not lead N225					
Total	0.554**	0.059**	0.058**	0.054**	0.056**	0.049*	0.049*	0.038
Sub-period 1	0.420**	0.064	0.062	0.070	0.062	0.085	0.068	0.086*
Sub-period 2	0.706**	0.092	0.078	0.110	0.095	0.086	0.088	0.070
Sub-period 3	0.719**	0.114*	0.116*	0.072	0.102	0.087	0.075	0.066
Sub-period 4	0.346**	0.093	0.091	0.102*	0.067	0.076	0.072	0.072
Sub-period 5	0.589**	0.109**	0.087	0.093*	0.086	0.078	0.088*	0.094*

+, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; *, p≤0.05, **, p≤0.01

The contemporaneous Brownian distance correlation is considerably lower than that between current VIX returns and the lagged (by one) S&P 500 returns in all periods considered providing, thus, more evidence in favour of the leverage relative to representativeness and affect heuristics hypothesis. Finally, no clear pattern appears to exist between the log implied volatility level in a given sub-period and the values of the respective distance correlations at the various lags.

For the pair (DAX 100, V1XI), the distance correlations involving return series with strictly positive lags, point to bi-directional causality (especially for the total, the third, and the fifth sub-period). The contemporaneous distance correlation is very high relative to those involving one lag in the DAX 100 or in the V1XI returns. The data, therefore, provide very strong support to the representativeness and affect heuristics hypothesis relative to the competing ones. The estimations results for the pair (N225, JNIV) are quality-wise similar to those for the pair S&P 500 and VIX with regard to leverage vs volatility feedback hypothesis; they, however, favour the representativeness and affect heuristics relative to the leverage hypothesis.

As noted in the Introduction, the measure (Brownian distance correlation) obtained through the E-statistics is general, in the sense that it captures both linear and non linear co-movement. Standard measures of association such as the Person correlation coefficient and standard tests of causality such as the Granger one assume that the underlying relationships are linear. It would be certainly interesting to investigate whether the linear and the more general approaches to co-movement and causality lead to similar results.

Table 3 presents the values of the Pearson correlation coefficient for contemporaneous changes in the stock and the "fear gauge" indices. The results are consistent with what is reported in Table 2 (first column); higher, in absolute value terms, Pearson correlation coefficients are associated with higher Brownian distance correlation coefficients. Moreover, both measures suggest that the strongest contemporaneous association is the one between the DAX and the V1XI and the weakest between the SP500 and the VIX.

Table 3: Pearson Correlation Coefficients

Period	SP500 and VIX	DAX and V1XI	N225 and JNIV
Total	-0.193**	-0.736**	-0.561**
Sub-period 1+	-0.101**	-0.829**	-0.348**
Sub-period 2	-0.084**	-0.637**	-0.688**
Sub-period 3	-0.322**	-0.827**	-0.761**
Sub-period 4	-0.318 **	-0.829**	-0.422**
Sub-period 5	-0.319**	-0.747**	-0.634**

+, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; **, p≤0.01

Table 4 present the results of the linear Granger causality tests. The null hypothesis that changes in the SP500 do not lead changes in the VIX is strongly rejected. The null, hypothesis, however, that changes in the VIX do not lead changes in the SP500 is consistent with the real world data in all but one sub-periods. The Granger test, therefore, points to uni-directional causality whereas the Brownian correlation coefficient (Table 2(a)) has largely pointed to a bi-directional one. The null hypothesis that changes in the DAX do not lead changes in the V1XI is rejected for three sub-periods (but not for the total period). There is no period, however, in which changes in the V1XI lead those in the DAX. Here, again, the Granger test offers some evidence of uni-directional causality whereas the Brownian motion correlation coefficient has pointed to a largely bi-directional one (Table 2 (b)). Very similar are the results of the Granger test for the pair N225 and JNIV.

Another notable difference between the results in Tables 2 and 4 is that whereas the non linear measure detects quite a few statistically significant associations at 3, 4, and (in certain cases) even at 7 lags, the Granger test indicates that the effect of shocks is very short-lived (the optimal lag length is everywhere less than or equal to 2). The fact that the Brownian correlation coefficient suggests that the effect of shocks has potentially a considerable duration whereas the Granger test indicates that the effect of shocks dies out very quickly must be attributed to the assumptions underlying the two approaches. The standard Granger test captures linear relations only; the Brownian correlation coefficient, however, works equally well with linear and non linear linkages (Creamer and Creamer, 2016).

Table 4: Granger Causality Tests (F Values)

Period	SP500 does not lead VIX	DAX 100 does not lead V1XI	N225 does not lead JNIV
Null hypothesis:			
Total	683.136** (2)	1.604 (1)	0.142 (1)
Sub-period 1+	362.527** (2)	3.937* (1)	6.013* (1)
Sub-period 2	366.541** (1)	8.626** (1)	0.366 (1)
Sub-period 3	220.076** (1)	1.618 (1)	2.116 (1)
Sub-period 4	87.226** (2)	0.007 (1)	3.765 (1)
Sub-period 5	206.044** (2)	4.848* (1)	4.718* (1)
Null hypothesis:			
	VIX does not lead SP500	V1XI does not lead DAX100	NJIV does not lead N225
Total	2.028 (2)	0.006 (1)	0.027 (1)
Sub-period 1+	1.535 (2)	1.260 (1)	0.001 (1)
Sub-period 2	13.45** (1)	0.006 (1)	0.388 (1)
Sub-period 3	0.863 (1)	0.228 (1)	13.963** (1)
Sub-period 4	1.763 (2)	0.935 (1)	2.870 (1)
Sub-period 5	1.651 (2)	0.441 (1)	2.340 (1)

+, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; *, p≤0.05, **, p≤0.01; optimal number of lags in parentheses, determined using the Bayesian Information Criterion (BIC).

4. Conclusions

In this study we have employed the Brownian distance correlation coefficient to investigate empirically the validity of three competing hypotheses (leverage, time-varying risk premium, and representativeness and affect heuristics) with regard to the contemporaneous and the lag-lead linkages between stock market and implied volatility indices. For the empirical analysis we have utilized daily observations over 2004 to 2017 from the S&P 500, the DAX 100, the N225, the VIX, the D1XI, and the JNIV.

The empirical results appear to provide strong support to the leverage relative to the volatility feedback hypothesis for the pairs (S&P 500, VIX), and (N225, JNIV). This is in line with the findings of Bollerslev et al. (2006). For the pair (DAX 100, V1XI), and in accordance with what has been reported by Chiang (2012), the evidence points to a bi-directional causality.

The contemporaneous Brownian correlations between stock market and volatility returns have received much higher values relative to those involving lags for the pairs (DAX 100, V1XI) and (N225, JNIV). This is consistent with the findings of Badshah (2013) and Hibbert et al. (2008) and favours the representativeness and affect heuristics hypothesis relative to the leverage and the time-varying risk premium hypotheses. For the pair (S&P 500, VIX), the evidence favours a lead-lag relation over a contemporaneous one.

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AN EMPIRICAL STUDY OF REGIONAL MUTUAL FUNDS' DIVERSIFICATION VALUE

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Abstract: This article studies three samples of United States-based regional mutual funds from the Asia-Pacific, Europe, and Latin America, to assess whether higher fund diversification translates into higher diversification values to fund shareholders. To measure mutual funds' portfolio diversification, we implement a modified Herfindahl index. To assess diversification values we employ a methodology that considers the Sharpe ratio of funds and its correlation with existent portfolios. We find that Asian-Pacific funds are the most diversified, whereas European funds provide the highest diversification value to fund shareholders. The correlation between fund diversification and diversification value is positive only in the case of Asian-Pacific funds.

Keywords: regional mutual funds; diversification value; portfolio diversification; Herfindahl index

1. Introduction

Business literature praises international diversification. Early studies show that United States (US) investors can attain a high diversification value by investing in emerging markets (Harvey, 1995), multinational firms (Rowland & Tesar, 1998), country funds, and American Depository Receipts ("ADRs") (Errunza, Hogan & Hung, 1999). Despite higher market integration and a reduction of investment barriers, international diversification values are still significant. Driessen and Laeven (2007) report that there exist significant diversification benefits for investors in both, developed and developing countries. However, these benefits are larger for investors in developing countries. Chiou (2009) demonstrates that even after monitoring portfolio constraints, international investments could generate economic value.

In their quest for international diversification, US investors may use securities issued by foreign corporations. However, this practice may not be cost-effective due to the capital required to adequately diversify their portfolios across many investments in the region. Additionally, some foreign markets are not even accessible to individual investors. Investors may also indirectly invest in foreign markets through investment companies. The four most common types of investment companies in the United States are open-end mutual funds, exchange traded funds, closed-end mutual funds, and unit investment trusts; where open-end mutual funds are the most widespread.

In its 2014 annual report, the Investment Company Institute ("ICI") stated that total net assets in mutual funds amounted to over $15 trillion. Whereas assets in exchange traded funds, closed-end mutual funds, and unit investment trusts totaled $1.7 trillion, $279 billion, and $87 billion respectively. In fact, 46.3 percent of all US households own an US-based open-end mutual fund, which suggests that they are the main vehicle where investors gain access to international markets. In 2013, international mutual funds' assets reached $2.1 trillion or 14 percent of the US mutual funds industry's total assets.

US-based international mutual funds include geographically speaking, well-diversified funds, as well as strictly constrained funds. For instance, foreign funds primarily invest in foreign securities while maintaining a limited amount of assets in the US, whereas regional funds manage portfolios with securities from a particular geographical region. Regional mutual funds usually invest in at least 80 percent of their portfolios in securities from a certain geographical area.

An under-researched issue that is central to this investigation is the analysis of regional funds' diversification value to fund shareholders. In addition to good performance, investors may benefit from adding mutual funds to their portfolios if new funds increase investors' overall diversification. The higher the diversification, the smoother or less volatile inventors' overall investment portfolio returns will be.

In this study, we examine the diversification value of US-based regional mutual funds that invest in Asia-Pacific, Europe, and Latin America (the "Study Regions'"). We study the funds' diversification value by analyzing their exposure across countries in their region and determining whether these funds' diversification benefits fund shareholders. Specifically, we ask the following question: does higher portfolio fund diversification translate into better diversification to fund shareholders? To the best of our knowledge, this issue has not yet been addressed in the literature pertaining to US-based regional mutual funds.

2. Literature Review

The literature on US-based regional mutual funds is quite limited. Some studies on United international mutual funds' risk-adjusted performance include regional funds as a sample (Babalos, Mamatzakis & Matousek, 2015; Basu & Huang-Jones, 2015; Tkac, 2001). Regarding European funds, the literature is constrained to a few studies that are solely devoted to these funds (Engstrom, 2003; Pushner, Rainish & Coogan, 2001; Papadamou & Stephanides, 2004; Rodriguez, 2008). For instance, Engstrom (2003) addresses European mutual funds' diversification value for international investors. Pushner, Rainish, & Coogan (2001) study European funds' performance during 1986 to 1998, finding that their sample underperformed when benchmarked with the MSCI European Index. Papadamou and Stephanides (2004) examine European mutual funds from a risk management perspective. Implementing various versions of Value at Risk ("VAR") and expected tail loss models, they find that either models' efficacy primarily depends on funds' investing style. Rodriguez (2007), however, focus on European mutual funds' forecasting ability by examining attribution returns, finding evidence of positive performance and good forecasting skill.

Many studies on emerging markets' mutual funds include Latin American funds as part of their samples (Borensztein & Gelos, 2003; Kaminsky, Lyons & Schumukler, 2001). Kaminsky, Lyons, and Schumukler (2001) is one of the few studies which are solely devoted to these mutual funds. They analyze a sample of open-end Latin American mutual funds and present momentum trading by both investors and fund managers. They also find contagion trading, like the systematic selling (or buying) of stocks in one country when the stock market falls (or rises) in another. Rodriguez (2007) study Latin

American funds' forecasting abilities during 1999 to 2003, to find good forecasting ability and positive risk-adjusted performance; which are saved for crises wherein forecasting ability is quite poor.

Only a few studies focus on Asia-Pacific mutual funds. For instance, DeMasky, Dellva, and Heck (2003) study the efficiency and effect of hedging currency risk by United States-based Asia-Pacific funds, showing that hedging improves these funds' risk-adjusted performance.

3. Data and Methodology

3.1 Data

This study focuses on United States-based Asia-Pacific, European, and Latin American mutual funds' diversification value during 2004 to 2014 (the "Study Period"). The samples include US-based Asian-Pacific, European, and Latin American mutual funds as identified in the Center for Research in Security Prices Survivorship-Bias-Free U.S. Mutual Fund Database ("CRSP"). We extracted funds' data as well as monthly returns from CRSP. For fund families with multiple classes of the same fund, that is, the same portfolio, we only include the fund class with the longest history in the sample. To be included in the study, a fund must have had at least 36 consecutive months of return data. To avoid the survivorship bias problems presented in Elton, Gruber, and Blake (1996), we include surviving and non-surviving funds in all analyses.

Table 1 provides the samples' descriptive statistics. The samples are 21 Asian-Pacific, 31 European, and 11 Latin American funds (each referred to as the "Asian-Pacific Sample," the "European Sample," and the "Latin American Sample," respectively, and collectively as the "Samples"). Based on median values, the European Sample contains the most total net assets (107.7 million), followed by the Asian-Pacific (36.43 million) and Latin American (27.14 million) Samples. Concerning expense ratio, the Latin American Sample exhibits the largest median value (1.64 percent), followed by the Asian-Pacific (1.58 Percent) and European (1.49 percent) Samples. Comparing Samples' median turnover ratio, the European Sample has the highest (88.3 percent), followed by the Asian-Pacific (74.2 percent) and Latin American (53.6 percent) Samples.

Table 1: Fund Samples Descriptive Statistics

Panel A: Asia-Pacific (21 funds)					
	Mean	**Std. Dev.**	**Median**	**Minimum**	**Maximum**
Total net assets	309.3438	606.2064	36.4375	0.675	2423.264
Expense Ratio	0.0164	0.0048	0.0158	0.009	0.0252
Turnover Ratio	0.7584	0.4216	0.7418	0.1763	1.7743

Panel B: Europe (31 funds)					
	Mean	Std. Dev.	Median	Minimum	Maximum
Total net assets	281.0163	447.7072	107.7	2.15	2178.618
Expense Ratio	0.014872	0.0044	0.0149	0.0084	0.0275
Turnover Ratio	1.1082	1.3337	0.8827	0.0563	7.78

Panel C: Latin America (11 funds)					
	Mean	Std. Dev.	Median	Minimum	Maximum
Total net assets	508.214	913.5466	27.14	0.7333	2701.473
Expense Ratio	0.0163	0.003	0.0164	0.0105	0.0221
Turnover Ratio	0.6805	0.5773	0.5364	0.1033	2.27

To estimate the various metrics employed in this study, we extracted country indexes' monthly returns from Morgan Stanley Capital International Index ("MSCI") through Bloomberg. In the end, we included a total of 29 MSCI country indexes in the ensuing analysis. To estimate the cash portion of funds' portfolios, we use the Fama-French risk-free rate.[1] We include the risk-free rate for each Sample as funds' cash holdings.

3.2 Methodology

To measure mutual funds' portfolio diversification across countries in the region, we implement a modified Herfindahl index (Woerheide & Persson, 1993). Out of five metrics used by Woerheide and Persson (1993) to measure unevenly distributed stock portfolios' diversification, the Herfindahl index was the most effective. Although mainly applied to measure the concentration of companies within an industry, the Herfindahl index has proved quite versatile. For instance, Hayden, Porath, and Westernhagen (2007) use it to measure portfolio diversification of individual loans of German banks , and more recently Cressy, Malipiero, and Murani (2014) utilize it to study venture capital firms' portfolios. In this study, we define the modified Herfindahl as:

$$DI = 1 - HI = 1 - \sum_{i=1}^{n} w_i^2$$

Where:

DI = diversification index or a measure of mutual funds' diversification;

HI = Herfindahl index; and

w = exposure to each country in the region where funds invest.

DI ranges between zero and one. The larger the value, the larger funds' diversification.

[1] Available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

We implement Sharpe's (1992) style analysis to estimate portfolio exposure to countries in each geographical region based on publicly available daily fund returns.
To implement Sharpe's style analysis, we express fund returns as:

$$r_i = \sum_{j=1}^{n} w_{i,j} r_j + e_i \qquad (1)$$

Where:

r_i = total return of fund *i;*

$w_{i,j}$ = exposure of fund *i* to country index *j;*

r_j = total return of country index *j;* and

e_i = unexplained component of funds' returns.

The portfolio weights are the solution of a quadratic programming problem. These weights represent factor loadings on an index strategy that best explains funds' return:

$$\mathrm{Min}\left[\mathrm{var}\left(\mathrm{r_i} - \sum_{j=1}^{n} w_{i,j} r_j\right)\right] \qquad (2)$$

subject to:

$$1 \leq w_{i,j} \leq 0 \quad \forall j$$

$$\sum_{j=1}^{n} w_{i,j} = 1$$

Style analysis helps compute active fund managements' value.[2] All countries we include in the style analysis are also included in each MSCI regional index. The Asian-Pacific countries included are: Australia, China, Hong Kong, India, Indonesia, Japan, Malaysia, New Zealand, Singapore, South Africa, South Korea, Taiwan, and Thailand; the European countries are: Austria, Belgium, Denmark, France, Germany, Italy, Spain, Sweden, Switzerland, and the United Kingdom; and the Latin American countries are: Argentina, Brazil, Chile, Colombia, Mexico, and Peru.

After estimating portfolios' fund diversification, we gauge diversification values provided to fund shareholders. To that end, we employ a methodology first introduced by Elton, Gruber, and Rentzler (1987). The underlying assumption for their approach is that a mutual fund should be added to an existing portfolio if its Sharpe ratio exceeds the product of the return correlation of the mutual fund with the existing portfolio and the Sharpe ratio of the existing portfolio. Namely, a mutual fund should be added to an existent portfolio if the following condition holds:

[2] Examples include Dor et al. (2003), Comer (2006) and Rodríguez (2008).

$$\frac{\overline{r_i} - r_f}{\sigma_i} > \left(\frac{\overline{r_P} - r_i}{\sigma_P} \right) \rho_{iP} \tag{3}$$

Where:

$\overline{r_i}$ = fund's average monthly return,

r_f = monthly risk-free rate,

σ_i = standard deviation of fund *F*,

$\overline{r_P}$ = average monthly return of the existing portfolio,

σ_P = standard deviation of portfolio *P*; and

ρ_{iP} = correlation coefficient between fund *i* and portfolio *P*.

We measure diversification value provided to fund shareholders as the difference between the ratios (left minus right).

Polwitoon and Tawatnuntachai (2006) and Shen, Lu, and Lin (2012) also study mutual funds' diversification value by implementing Elton et al.'s (1987) methodology. The former examined global bond funds, whereas the latter considered international real estate mutual funds. Following Polwitoon and Tawatnuntachai's (2006) approach to examine regional funds' incremental diversification value, we utilize index funds to represent typical portfolios of United States-based mutual fund investors. Index funds rather than index benchmarks, represent a better proxy of investors' portfolios, as funds account for expenses. To measure the portfolio of a typical United States investors we use Vanguard 500 index mutual fund.

4. Empirical Results

First, we estimate funds' exposure to all countries in each study region during the study period. Table 2 shows these results. Panel A of this table shows the Asian-Pacific Sample's average exposure. These funds exhibit the highest exposure to Japan (14.08 percent), followed by Hong Kong (13.93 percent), and Thailand (13.86 percent). Panel B presents the European Sample's average exposure. This Sample is primarily exposed the United Kingdom (23.56 percent), Germany (23.13 percent), and Austria (20.64 percent). Panel C shows the Latin American Sample's exposure. This Sample is mainly exposed to Brazil (48.47 percent), Mexico (28.85 percent), and Colombia (6.16 percent). Table 2 includes the adjusted R2 for the Sharpe estimation, indicating that this estimation was effective for all three Samples as it explains between 92 and 99 percent of regional mutual funds' return variation.

We now turn to the crux of the study. We estimated regional mutual funds' diversification value via a modified Herfindahl index. Table 3 provides descriptive statistics of fund diversification and diversification value provided to fund shareholders. Panel A shows the Asian-Pacific Sample results. The Asian-Pacific Sample's portfolio diversification is high as the average and median DI (diversification index) are 0.8193 and 0.8237, respectively. However, the average diversification value provided to fund shareholders is -0.0359, meaning that, on average the Asian-Pacific Sample failed to provide diversification value to shareholders. Moreover, only nine funds of this Sample provided diversification value to fund shareholders; that is, only nine funds exhibited a positive Elton et al. diversification measure (Equation 3; the "Diversification Measure"). Finally, we find a low, but positive correlation (0.3214) between funds' diversification and the diversification value provided to Asian-Pacific shareholders during the Study Period, suggesting that higher fund diversification translates to a higher diversification value to fund shareholders.

Table 2: Mutual Funds Country Exposure

Panel A: Asia-Pacific		Panel B: Europe		Panel C: Latin America	
Country	**Exposure**	**Country**	**Exposure**	**Country**	**Exposure**
Australia	6.44%	Austria	20.64%	Argentina	1.01%
China	11.46%	Belgium	4.93%	Brazil	48.47%
Hong Kong	13.93%	Denmark	1.00%	Chile	5.87%
India	4.29%	France	3.40%	Colombia	6.16%
Indonesia	6.95%	Germany	23.13%	Mexico	28.85%
Japan	14.08%	Italy	2.18%	Peru	3.59%
Malaysia	0.56%	Spain	12.47%	Cash	6.05%
New Zealand	0.00%	Sweden	6.87%		
Singapore	10.71%	Switzerland	0.00%		
South Africa	6.65%	United Kingdom	23.56%		
South Korea	3.90%	Cash	1.81%		
Taiwan	5.49%				
Thailand	13.86%				
Cash	1.67%				
Ave. Adjusted r2	0.95	Ave. Adjusted r2	0.92	Ave. Adjusted r2	0.99

Table 3, Panel B presents the European Sample results. The average and median fund diversification are 0.737 and 0.7518, respectively. Overall, European funds offered diversification value, as the average diversification value to fund shareholders is 0.0086. Also, 15 out of 31 European funds showed a positive Diversification Measure. However, we find that high fund diversification means lower diversification value to fund shareholders, as the correlation between these two measures is -0.1425.

Finally, Panel C shows the Latin American Sample results. The average fund diversification (DI) is 0.6005, whereas the median is 0.5848. Regarding diversification value to fund shareholders, this sample fell short as its average Diversification Measure is -0.1157, and only five funds provided diversification value to fund shareholders. As in the European Sample, the correlation between fund diversification and diversification value to fund shareholders is negative (-0.612).

Table 3: Mutual Fund Diversification and Diversification Value To Investors

Panel A: Asia-Pacific (21 funds)	Mean	Std. Dev.	Median	Minimum	Maximum
Fund Diversification	0.8193	0.0612	0.8237	0.6474	0.8868
Diversification Value	-0.0359	0.1156	-0.0203	-0.2686	0.2359
Correlation	0.3214				

Panel B: Europe (31 funds)	**Mean**	**Std. Dev.**	**Median**	**Minimum**	**Maximum**
Fund Diversification	0.7370	0.0625	0.7518	0.5020	0.8344
Diversification Value	0.0086	0.0985	-0.0051	-0.2011	0.2538
Correlation	-0.1425				

Panel C: Latin America (11 funds)	**Mean**	**Std. Dev.**	**Median**	**Minimum**	**Maximum**
Fund Diversification	0.6005	0.0843	0.5848	0.4916	0.7368
Diversification Value	-0.1157	0.2090	-0.0501	-0.4522	0.2182
Correlation	-0.6120				

5. Conclusion

This study examines the diversification level of three Samples of US-based regional mutual funds, and diversification value these funds provided to fund shareholders. To measure fund diversification, we employ a modified Herfindahl index. To determine diversification value provided to fund shareholders we used a methodology based on Elton et al. (1987).

Results show that the Asian-Pacific Sample has the highest portfolio diversification, but does not provide diversification value to fund shareholders. Nevertheless, the correlation between fund diversification and diversification value provided to fund shareholders is positive. In the case of the European Sample, fund diversification is lower than that of the Asia-Pacific Sample, but diversification value provided to fund shareholders is higher. However, the correlation between the two is negative.

Overall, the Latin American Sample was the less diversified, and as the Asia-Pacific Sample, it did not to provide diversification value to fund shareholders. However, as in the case of the European Sample, the Latin American Sample's fund diversification is associated with lower diversification value to fund shareholders.[3]

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