

# OIL VOLATILITY-OF-VOLATILITY AND TAIL RISK OF COMMODITIES

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## Abstract

We examine the information content of oil volatility-of-volatility (VOV), constructed from the past 1-month OVX (implied volatility in crude oil market), on the expected tail risk of commodities proxied by Value at Risk (VaR) and Expected Shortfall (ES). Specifically, we find oil VOV predicts 1-step-ahead tail risks of Energy and the Aggregate Commodity sector (GSCI) for both in-sample and out-of-sample. Our results indicate the important role of crude oil in overall commodity markets by incorporating forward-looking information of OVX. Our findings are robust and complement the strand of literature about the leading role of crude oil in commodity markets.

**Keywords:** Commodity markets, volatility-of-volatility risk, expected tail risk

## 1. Introduction

Over the past decade, commodity markets have experienced substantial fluctuations. The availability and popularity of new commodity-linked securities, due to the financialisation of the commodity markets, have led to extraordinary shifts in return dynamics of commodities. An emerging literature has focused on understanding tail risk in commodity markets. Value at Risk (VaR) and Expected Shortfall (ES) are two well-known metrics used to quantify tail risk. Specifically, VaR measures the potential maximum loss of an investment at a certain confidence level over a specific time frame. In contrast, ES takes an advantage of sub-addition by considering the expected value of the loss of the portfolio below a certain confidence level and is more sensitive to the shape of the tail of loss distribution (e.g., Frey and McNeil, 2002). Both VaR and ES have been widely used as measures for tail risks, which is essential for asset pricing and risk management.

Among commodities, the crude oil market plays a crucial role in transmitting risk among commodity markets, such as precious metal markets (e.g., Ahmed et al., 2022; Reboredo & Ugolini, 2016; Shahzad et al., 2019), clean energy sectors (Foglia et al., 2022), and financial sectors (Zhao et al., 2022). The literature has demonstrated that volatility-of-volatility (VOV) is a significant state variable containing nonredundant pricing information of oil volatility. In this paper, we contribute to this strand

of research by providing evidence that VOV of oil market predicts tail risks of several other commodities, including energy and the aggregate commodity sector<sup>1</sup>.

Crude oil price plays an important role in commodity markets since crude oil is a major input for production, and therefore, its prices are closely related to the costs of production and consumption. For example, Tyner (2010) finds that higher crude oil prices lead to higher gasoline prices, and subsequently higher demand for corn ethanol, which finally causes higher corn and commodity prices. Baumeister and Kilian (2014) also identify evidence of higher prices of agricultural commodities due to the transmission of oil price shocks. Melichar and Atems (2019) demonstrate that oil-demand shocks serve as the main driver for higher commodity prices before 2006, whereas oil supply shocks show impacts after expanded ethanol production since 2006. Higher oil prices are closely related to increasing volatility and uncertainty of volatility (i.e., VOV). Thus, oil VOV, a measure of the uncertainty of implied oil volatility, is likely to have a major impact on future commodity prices.

We mainly investigate the predictability of oil VOV on the future tail risk, proxied by VaR and ES, of the aggregate commodity market and its five subsectors, namely, energy, precious metal, industrial metal, agriculture, and livestock. Specifically, oil VOV shows significant predictability for the energy sector and the aggregate commodity market, using both tail risk measures of VaR and ES. These findings are consistent with previous studies that tail risk of oil market spillovers to other commodity markets such as metals and other energy sectors. Our results are robust after controlling for other volatility-related variables of equity and crude oil markets and fundamental economic variables. In addition, we perform out-of-sample tests by employing several statistics including out-of-sample R-square ( $R_{os}^2$ ), McCracken's (2007) F-statistic (MSE-F), and ENC statistic proposed by Clark and McCracken (2001) (ENC-NEW).

Our contributions are mainly two-folded. First, we identify an oil-market factor representing the uncertainty level of oil volatility that significantly improves the forecasting performance on the tail risk of commodity markets, whereas most previous literature has focused on the oil implied volatility and very limited discussions have so far been put forward about the role of oil VOV. Second, our results shed light on the linkage of tail risk between oil market and other commodities, by utilising the forward-looking information contained in oil VOV.

## 2. Data and key variables

### 2.1 Data

The empirical analysis covers the period of May 2007 to July 2021. We obtain all daily data from LSGE Datastream which includes volatility-related variables such as OVX, VIX, and VVIX and price variables from aggregate commodity market, precious metal sector, industrial metal sector, livestock sector, agriculture sector, and energy sector.

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<sup>1</sup> We proxy aggregate commodity market, energy, precious metal, industrial metal, agriculture, and livestock by using S&P GSCI Commodity, S&P GSCI Energy, S&P GSCI Precious Metal, S&P GSCI Industrial Metal, S&P GSCI Agriculture, and S&P GSCI Livestock, respectively.

## 2.2 Oil VOV

The oil VOV measure, denoted by  $vo_v_t^2$ , is computed based on the EWMA model as following:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) \mu_{t-1}^2, \quad (1)$$

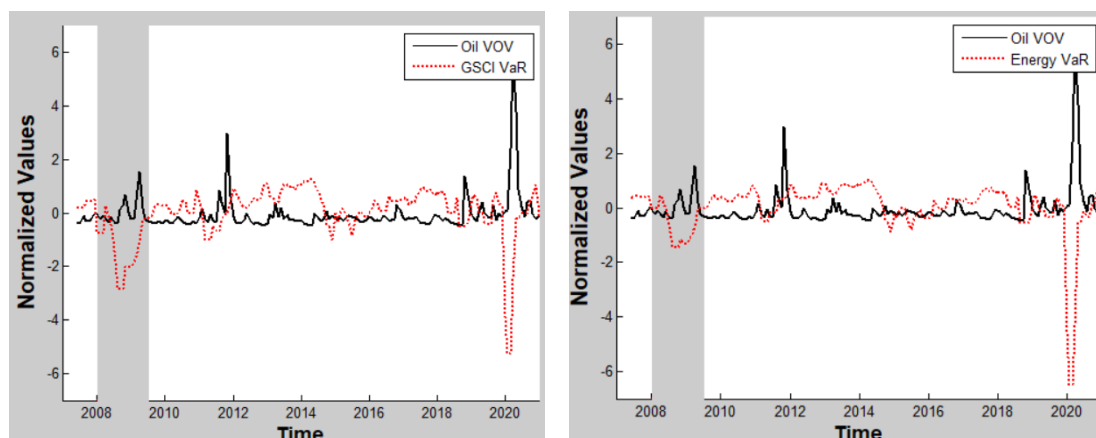
where  $\mu_t$  is the logarithmic return of OVX,  $\sigma_t$  denotes the conditional volatility of the gross return of the OVX, and  $\lambda$  measures the degree of the weighting decrease, set with the value of 0.94.<sup>2</sup>

## 2.3 Tail risk measure

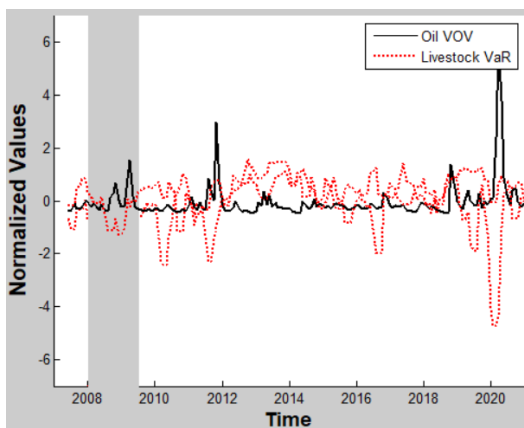
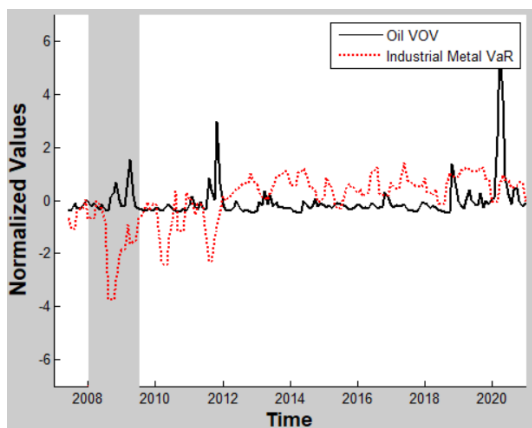
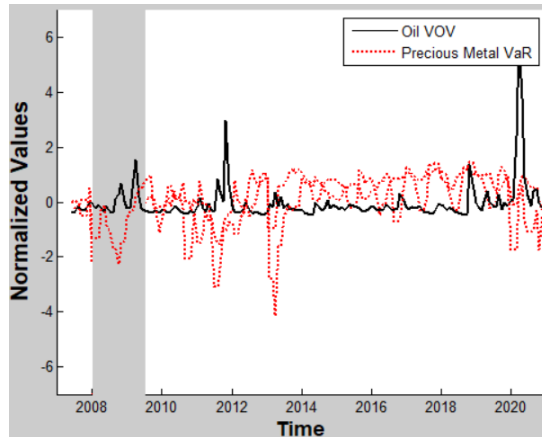
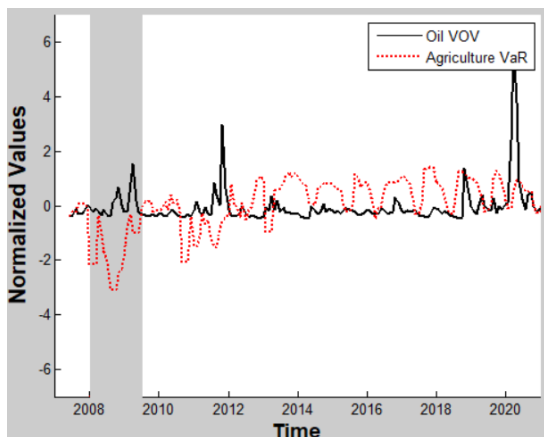
We consider the two most commonly used tail risk measures, namely Value-at-risk (VaR) and Expected Shortfall (ES), where the former measures the potential risk of loss, or the largest value of the potential loss, and the latter measures the expected portfolio return in the left tails. Both VaR and ES are computed at risk level of 5%, by using historic 3-month daily returns, namely, historical simulation (HS). Compared to other computation methods, this method is simpler and more straightforward (Christoffersen, 2003; Dowd, 2002; Kuester et al., 2006). The separate Appendix (Table A.1) presents the summary statistics for the key variables, namely, oil VOV, VaR, and ES at 5% level for the Aggregate commodity sector (GSCI), Energy, Precious Metals, Agriculture, and Livestock sector. Figure 1 shows the time-series plots of oil VOV and tail risk measure of each commodity sector with Panel A using VaR and Panel B using ES, respectively, which can provide clearer insights into how the variables interact with each other.

**Figure 1:** Oil VOV and Tail Risks of Commodities

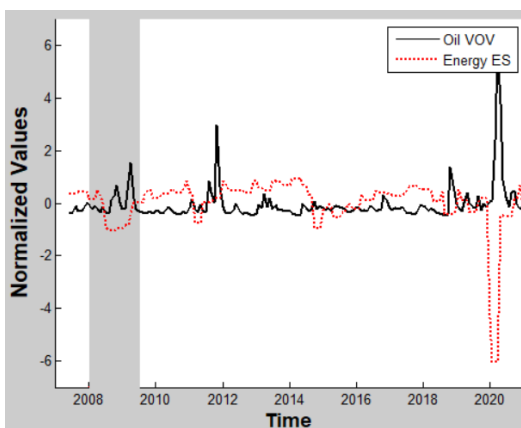
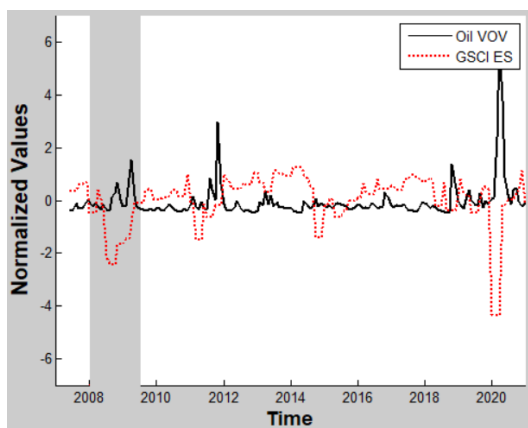
Panel A. Tail Risks Proxied by VaR

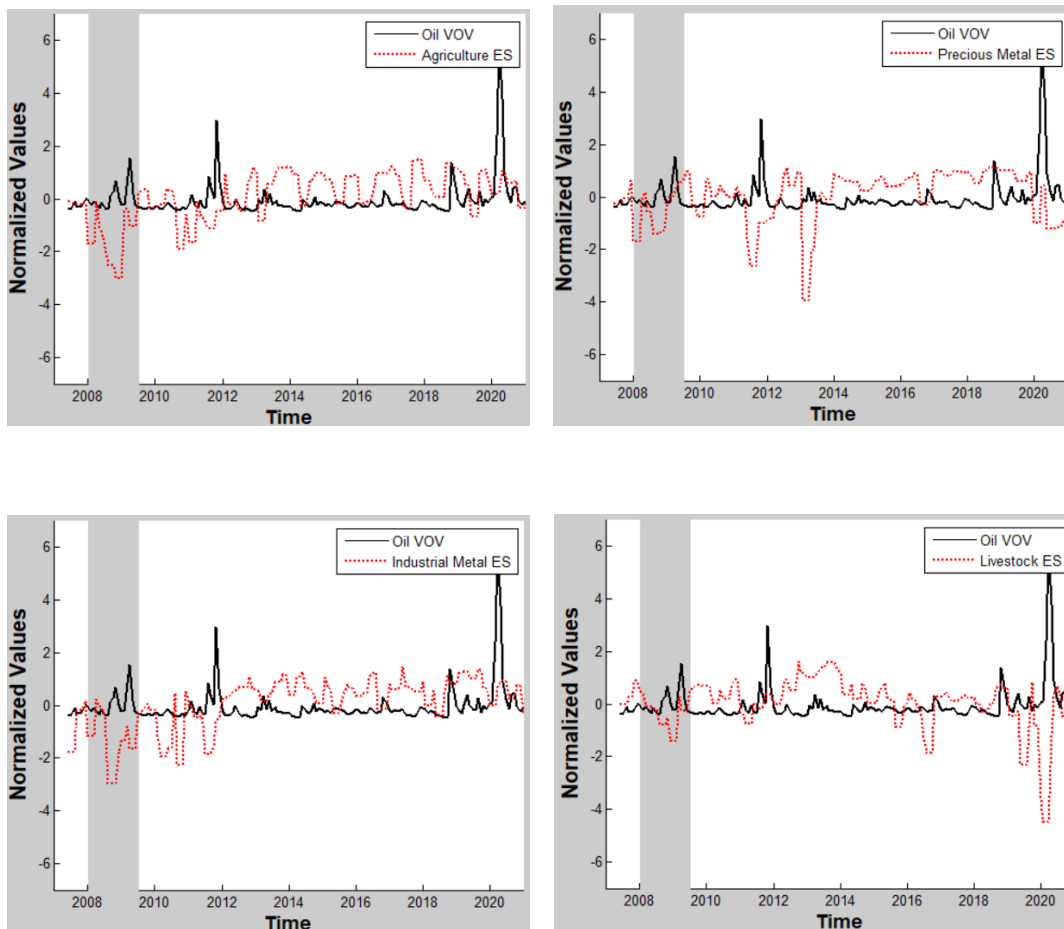


<sup>2</sup> We also use the standard deviation of the squared returns of log OVX as an alternative proxy for oil volatility of volatility, and the predictability of the proxy measure remains similar.



Panel B. Tail Risks Proxied by ES





Note: This figure shows the time-series plots of oil VOV and tail risk measures of commodities, including the aggregate commodity market (i.e., GSCI), energy, agriculture, precious metals, industrial metals, and livestock sector. Panel A and B are using tail risk measures proxied by VaR and ES, respectively. The sample period is from May 2007 to July 2021.

### 3. Empirical results

#### 3.1 In-sample predictability

We measure conditional higher-moment risks based on the following predictive regression:

$$Tail_{i,t+1} = \alpha + \beta_1 vov_t + \beta_2 Tail_{i,t} + \epsilon_{t+1}, \tag{2}$$

where  $Tail_{i,t+1}$  denotes commodity  $i$ 's tail risk for month  $t + 1$ , proxied by VaR or ES at 5% computed using returns from month  $t + 1$  to  $t + 3$  which is  $VaR_{t+3}$  or  $ES_{t+3}$  actually. We use  $vov_t$  with end of month  $t$  observations.

The results are reported in Table 1. Oil VOV shows negative and significant predictability for 1-period-ahead tail risks of the aggregate commodity market (i.e., GSCI), livestock, agricultural, and energy sector. In other words, an increase in oil VOV risk leads to higher downside risks for several other commodities and thus overall commodity market ultimately; our findings complement previous

findings about the uncertainty of the oil market (e.g., Asai et al., 2020; Ji & Fan, 2012; Nazlioglu et al., 2013). These patterns indicate evidence that tail risk spills over from the crude oil market to other non-oil commodities and highlights the leading role of the crude oil market (e.g., Ahmed et al., 2022; Reboredo & Ugolini, 2016; Zhao et al., 2022).

**Table 1: In-Sample Predictive Regression: Univariate Analysis**

	Panel A: VaR				Panel B: ES			
	Const.	vov	Lagged term	Adj-R <sup>2</sup> (%)	Const.	vov	Lagged term	Adj-R <sup>2</sup> (%)
GSCI	-0.003***	0.319***	0.918***	80.37	-0.004***	0.518***	0.921***	77.35
(t-stat)	(-3.27)	(7.14)	(19.02)		(-3.55)	(8.16)	(20.34)	
Precious Metals	-0.004***	0.083**	0.807***	64.02	-0.005***	0.059	0.816***	65.69
(t-stat)	(-3.68)	(2.25)	(14.18)		(-4.19)	(1.20)	(18.98)	
Industry Metals	-0.002***	0.055*	0.892***	79.52	-0.003***	0.079***	0.887***	78.61
(t-stat)	(-2.84)	(1.92)	(17.68)		(-3.63)	(2.63)	(21.50)	
Livestock	-0.002**	0.325***	0.946***	75.32	-0.002*	0.299***	0.947***	76.53
(t-stat)	(-2.02)	(3.72)	(11.04)		(-1.93)	(3.36)	(12.74)	
Agriculture	-0.002**	0.054*	0.896***	80.20	-0.003***	0.061	0.879***	76.94
(t-stat)	(-2.40)	(1.89)	(16.57)		(-3.08)	(1.46)	(19.86)	
Energy	-0.004***	0.520***	0.914***	76.93	-0.005***	1.275***	0.973***	78.36
(t-stat)	(-3.18)	(6.05)	(16.38)		(-3.00)	(6.80)	(15.96)	

Note: This table reports shows the 1-month ahead predictability of oil volatility-of-volatility (VOV) for 1-period-ahead tail risks of aggregate commodity market (i.e., GSCI), energy, precious metals, and livestock sector. We consider two tail risk measures: VaR (Panel A) and ES (Panel B) at 5% level. Newey and West (1987) robust t-statistics, significant at the 1%, 5%, and 10% levels, denoted respectively by \*\*\*, \*\*, and \*. The sample period is from May 2007 to July 2021.

The results are robust after controlling for other predictors, including oil market volatility, equity market volatility, equity market VOV, and a set of fundamental economic variables. The economic specification is as follows:

$$Tail_{i,t+1} = \alpha + \beta_1 vov_t + \beta_2 Tail_{i,t} + \beta_3 vol_t + \beta_4 VVIX_t + \beta_5 VIX_t + \gamma' x_t + \epsilon_{t+1}, \tag{3}$$

where vol is the oil market volatility, VVIX is the equity market, VIX is the equity market volatility, and x is the vector of fundamental economic variables including the term spread (i.e., TS), the default spread (i.e., DS), and the dividend-price ratio (i.e., DP). The results are presented in Table 2. The forecasting power of Oil VOV remains significant after controlling for oil volatility, equity VOV and fundamental economic variables. Notably, the oil volatility also shows significant predictability for the overall commodity market, and several individual sectors including livestock, agriculture, and energy. Our findings suggest that both volatility and tail risk spillovers from crude oil to other non-oil commodity markets. In sum, oil VOV contains unique information that cannot be covered by its equity counterpart and other volatility-related measures. Our findings highlight the prominent role of the crude oil market in disseminating information about economic conditions to other commodity markets.

**Table 2: In-Sample Predictive Regression: Controlling Other Predictors**

Panel A: VaR										
	Const.	vov	Lagged term	OVX(10 <sup>3</sup> )	VVIX(10 <sup>3</sup> )	VIX(10 <sup>3</sup> )	TMS	DEF	PD	Adj-R <sup>2</sup> (%)
GSCI	0.044	0.428***	1.018***	0.209*	0.061	0.040	0.102	-0.637**	-0.014*	75.18
(t-stat)	(1.45)	(4.60)	(11.89)	(1.73)	(1.16)	(0.39)	(1.16)	(-2.25)	(-1.90)	
Precious Metals	0.004	-0.010	0.768***	0.090	0.054	-0.215*	0.015	0.045	-0.004	60.58
(t-stat)	(0.13)	(-0.14)	(14.10)	(1.31)	(1.07)	(-1.79)	(0.24)	(0.20)	(-0.60)	
Industry Metals	0.044***	0.110**	0.838***	-0.023	0.097*	-0.099	-0.034	-0.168	-0.013***	77.28
(t-stat)	(2.66)	(2.12)	(12.63)	(-0.50)	(1.82)	(-1.07)	(-0.71)	(-1.10)	(-3.35)	
Livestock	0.003	0.314***	0.876***	-0.034	0.049*	-0.057	0.072**	0.013	-0.003	74.33
(t-stat)	(0.20)	(3.76)	(10.13)	(-1.04)	(1.79)	(-0.96)	(2.11)	(0.12)	(-0.69)	
Agriculture	0.058*	0.111**	0.737***	-0.025	0.202***	-0.298***	0.009	-0.201	-0.019**	73.40
(t-stat)	(1.65)	(2.54)	(10.86)	(-0.62)	(3.70)	(-2.76)	(0.20)	(-0.97)	(-2.19)	
Energy	0.069	1.425***	1.063***	0.389**	0.029	-0.009	0.184	-1.129**	-0.020*	77.67
(t-stat)	(1.52)	(7.87)	(11.86)	(2.14)	(0.40)	(-0.05)	(1.25)	(-2.15)	(-1.78)	
Panel B: ES										
	Const.	vov	Lagged term	OVX(10 <sup>3</sup> )	VVIX(10 <sup>3</sup> )	VIX(10 <sup>3</sup> )	TMS	DEF	PD	Adj-R <sup>2</sup> (%)
GSCI	0.025	1.004***	0.866***	-0.103	0.102*	-0.173	0.110	0.069	-0.010	67.09
(t-stat)	(0.65)	(5.42)	(17.25)	(-1.16)	(1.84)	(-1.45)	(0.99)	(0.25)	(-1.06)	
Precious Metals	0.023	-0.159*	0.766***	0.084	0.118*	-0.238	0.003	0.001	-0.010	60.86

( <i>t</i> -stat)	(0.52)	(-1.67)	(15.71)	(1.04)	(1.84)	(-1.47)	(0.04)	(0.00)	(-0.97)	
Industry Metals	0.065***	0.134**	0.677***	0.018	0.176**	-0.421**	-0.051	-0.225	-0.021***	65.76
( <i>t</i> -stat)	(2.87)	(2.09)	(8.10)	(0.33)	(2.00)	(-2.39)	(-0.71)	(-0.98)	(-3.40)	
Livestock	0.002	0.321***	0.814***	-0.061*	0.051	-0.083	0.083**	0.058	-0.003	68.92
( <i>t</i> -stat)	(0.09)	(2.88)	(9.44)	(-1.70)	(1.64)	(-1.18)	(1.97)	(0.38)	(-0.47)	
Agriculture	0.079*	0.261***	0.665***	-0.063	0.252***	-0.503***	0.008	-0.209	-0.025**	69.39
( <i>t</i> -stat)	(1.89)	(5.15)	(10.53)	(-1.45)	(3.66)	(-4.24)	(0.15)	(-0.82)	(-2.43)	
Energy	0.033	2.768***	0.972***	-0.090	0.067	-0.302	0.268	0.080	-0.012	74.32
( <i>t</i> -stat)	(0.54)	(4.37)	(10.04)	(-0.40)	(0.86)	(-1.32)	(1.36)	(0.15)	(-0.81)	

Note: This table reports shows the predictability of oil volatility-of-volatility (VOV) for 1-period-ahead tail risks of aggregate commodity market (i.e., GSCI), energy, precious metals, and livestock sector based on VaR (Panel A) or ES (Panel B) at 1% level. We control other predictors including oil market volatility (OVX), VIX, VVIX, the term spread (i.e., TMS), the default spread (i.e., DEF), and the dividend-price ratio (i.e., DP). The *t*-statistics are computed according to Newey and West (1987), significant at the 1%, 5%, and 10% levels, and denoted respectively by \*\*\*, \*\*, and \*. The sample period is from May 2007 to July 2021.

## 3.2 Robustness check

### 3.2.1 Other tail risk measures

In our main analysis, we use tail risk measures computed at the 5% level; therefore, we further check the in-sample predictability of VaR and ES computed at risk levels of 1%. The results are shown in Table 3. The tail risk spillovers can still be found from the crude oil market to other commodity markets such as energy, agriculture, livestock, and the overall commodity market, at a more extreme case. Our results indicate that when the economy faces extreme downside fluctuations, the crude oil market plays a prominent role in disseminating information about economic conditions to other commodity markets.<sup>3</sup>

<sup>3</sup> We also check the VaR and ES computed using historic 6-month daily returns, and the results remain robust. Details for the analysis will be available upon request.



**Table 3: In-Sample Predictive Regression: Other Risk Level (VaR or ES at 1% Level)**

Panel A: VaR										
	Const.	vov	Lagged term	OVX(10 <sup>3</sup> )	VVIX(10 <sup>3</sup> )	VIX(10 <sup>3</sup> )	TMS	DEF	PD	Adj-R <sup>2</sup> (%)
GSCI	0.044	0.428***	1.018***	0.209*	0.061	0.040	0.102	-0.637**	-0.014*	75.18
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Livestock	0.003	0.314***	0.876***	-0.034	0.049*	-0.057	0.072**	0.013	-0.003	74.33
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Panel B: ES										
	Const.	vov	Lagged term	OVX(10 <sup>3</sup> )	VVIX(10 <sup>3</sup> )	VIX(10 <sup>3</sup> )	TMS	DEF	PD	Adj-R <sup>2</sup> (%)
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(t-stat)	(0.65)	(5.42)	(17.25)	(-1.16)	(1.84)	(-1.45)	(0.99)	(0.25)	(-1.06)	
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### 3.2.2 Out-of-sample analysis

The in-sample predictability could be due to overfitting and thus might not imply out-of-sample predictability (Welch & Goyal, 2008). Thus, we conduct a group of statistical tests to assess the out-of-sample forecasting power of oil VOV. Following Campbell and Thompson (2008) and Rapach et al. (2010), the main measure we consider to assess out-of-sample forecasting performance is out-of-sample R-square ( $R_{OS}^2$ ).<sup>4</sup> Additionally, we calculate McCracken's (2007) F-statistic (MSE-F), ENC-NEW statistic proposed by Clark and McCracken (2001) to obtain statistical inferences for the out-of-sample forecasting performance.<sup>5</sup> Out-of-sample statistics are constructed based on rolling windows with initial lengths of 60 months.<sup>6</sup>

Out-of-sample results are reported in Table 4. We observe that the strong in-sample predictability of oil VOV remains out-of-sample for GSCI and energy, as indicated by positive  $R_{OS}^2$ , significant values of the MSE-F and ENC statistics at the 5% level. Overall, we conclude that oil VOV predicts near-term tail risks of GSCI, and energy both in- and out-of-sample analysis.

**Table 4: Out-of-Sample Test**

	Panel A : VaR			Panel B : ES		
	OOS-R <sup>2</sup> (%)	MSE-F	ENC-NEW	OOS-R <sup>2</sup> (%)	MSE-F	ENC-NEW
GSCI	3.033	3.472***	25.066***	6.555	7.787***	6.441***
Energy	22.957	33.076***	30.360***	10.083	12.447***	10.124***
Precious Metals	-10.844	-10.859	3.671**	-6.343	-6.621	0.548
Industry Metals	-7.078	-7.338	6.492***	0.200	0.222	1.034
Agriculture	-0.224	-0.248	1.108	-6.064	-6.346	-2.054
Livestock	-7.962	-8.186	-2.660	-21.478	-19.625	-6.969

Note: This table reports shows the out-of-sample forecasting power of oil volatility-of-volatility (VOV) for 1-period-ahead tail risks of aggregate commodity market (i.e., GSCI), energy, precious metals, and livestock. We consider the following out-of-sample performance metrics: Out-of-sample  $R^2$  (OOS-R<sup>2</sup>), McCracken's (2007) F-statistic (MSE-F), and ENC-NEW statistic proposed by Clark and McCracken (2001). MSE-F and ENC-NEW, significant at the 1%, 5%, and 10% levels, and denoted respectively by \*\*\*, \*\*, and \*. Out-of-sample statistics are constructed based on rolling windows with initial lengths of 60 months. The sample period for out-of-sample test is from May 2012 to July 2021.

<sup>4</sup> A positive  $R_{OS}^2$  suggests that the predicted model outperforms the historical average benchmark.

<sup>5</sup> Details for computation of the statistics can be found in Appendix.

<sup>6</sup> The rolling scheme is robust to structural changes or regime shifts.

#### 4. Conclusion

In this paper, we find that oil VOV significantly predicts tail risks of the energy sector and the aggregate commodity market. The forecasting power of oil VOV remains robust after controlling for a set of predictors, including oil market volatility, equity market volatility, equity market VOV, and a set of fundamental economic variables. Notably, both oil volatility and VOV show significant predictability for several individual and aggregate commodity markets, highlighting the leading role of crude oil in commodity markets. Our findings are important for risk management and portfolio selection in commodity markets. More specifically, investors could obtain an optimal portfolio that effectively manages tail risks when investing in commodity markets, and this is mostly relevant during financial turmoil.

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## Appendix

### Out-of-Sample Evaluation Measures

$R_{OS}^2$  measures the proportional reduction in the mean squared error for the OLS model with the predictor relative to the model excluding the predictor only.  $R_{OS}^2$  is computed as,

$$R_{OS}^2 = 1 - \frac{MSE_A}{MSE_N}$$

where  $MSE_A = \frac{1}{T} \sum_{t=1}^T e_{A_t}^2$  denotes the mean squared error for the OLS model with the predictor and  $MSE_N = \frac{1}{T} \sum_{t=1}^T e_{N_t}^2$  denotes the mean squared error for the model excluding the predictor.  $T$  is the number of observations of the out-of-sample regressions.

The McCracken's (2007)  $F$ -statistic (MSE-F) is designed to test statistically whether an unrestricted model (models with the predictor) can beat a restricted model (the model excluding the predictor) in terms of out-of-sample forecasting performance. This measure is calculated as,

$$MSE - F = T \times \left( \frac{MSE_N - MSE_A}{MSE_A} \right)$$

We use the critical values derived by McCracken (2007) to obtain statistical inference for the MSE-F statistics. Another measure that we consider is ENC, which was also designed as a statistical test and proposed by Clark and McCracken (2001):

$$ENC = T \times \left( \frac{\sum_{t=1}^T e_{N_t}^2 - e_{N_t} \cdot e_{A_t}}{MSE_A} \right)$$

The critical values shown in Clark and McCracken (2001) are used to obtain statistical inference.

**Table A.1: Summary Statistics**

	Mean	SD	Min	Max	Skew	Kurt
Panel A: Oil VOV	0.003	0.006	4.24e-4	0.070	8.229	85.271
Panel B: VaR (5%)						
GSCI	-0.032	0.018	-0.125	-0.011	-2.615	12.387
Precious Metals	-0.029	0.012	-0.080	-0.011	-1.272	4.836
Industry Metals	-0.029	0.012	-0.075	-0.011	-1.521	5.661
Livestock	-0.021	0.008	-0.057	-0.008	-2.204	10.743
Agriculture	-0.028	0.012	-0.065	-0.010	-1.039	3.681
Energy	-0.046	0.033	-0.257	-0.013	-4.222	25.558
Panel C: ES (5%)						
GSCI	-0.037	0.020	-0.125	-0.012	-2.243	9.592
Precious Metals	-0.034	0.017	-0.101	-0.014	-1.560	5.953
Industry Metals	-0.033	-0.077	-0.012	0.015	-1.141	3.747
Livestock	-0.023	0.009	-0.062	-0.009	-2.030	9.477
Agriculture	-0.032	0.014	-0.075	-0.010	-0.837	3.379
Energy	-0.054	0.041	-0.302	-0.015	-4.332	25.613

Note: This table reports descriptive statistics such as the mean (Mean), standard deviation (SD), minimum (Min), maximum (Max), skewness (Skew), and kurtosis (Kurt) for oil VOV, VaR and ES at 5% level for the aggregate commodity sector (GSCI), Energy, Precious Metals, Industry Metals, Agriculture, and Livestock sector. The sample period is from May 2007 to July 2021.