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CAUSALITY BETWEEN STOCK MARKET AND "FEAR GAUGE" INDICES: AN EMPIRICAL ANALYSIS WITH E-STATISTICS

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Abstract: This study investigates empirically the validity of three hypotheses that have been advanced to explain the tendency of stock market and volatility indices to move in opposite directions, using the notion of Brownian distance correlation. We consider three stock market-implied volatility index pairs, namely, the S&P 500 and the VIX, the DAX 100 and the V1XI, and the N225 and the JNIV. The empirical results support the leverage hypothesis relative to the volatility feedback hypothesis for the pairs S&P 500 and VIX, and N225 and JNIV, and the representativeness and affect heuristics hypothesis relative to the leverage hypothesis for the pairs DAX 100 and V1XI, and N225 and JNIV.

Keywords: Brownian distance, stock index, volatility index

1. Introduction

The negative correlation between stock market and volatility returns has been well documented in Finance literature suggesting a potential diversification benefit to including volatility in an investment portfolio (e.g. Badshah, 2013; Bollerslev et al., 2006; Whaley, 1993; Campbell and Hentschel, 1992; Black, 1976). At the same time, however, there is a little agreement among researchers concerning the mechanism behind the tendency of stock market indices and volatility indices to move in opposite directions. Leading explanations include the leverage hypothesis (Christie, 1982; Black, 1976), the volatility feedback or time-varying risk premium hypothesis (Campbell and Hentschel, 1992; French et al., 1987), and the representativeness and affect heuristics hypothesis (Badshah, 2013; Hibbert et al., 2008). The first attributes the negative relationship between stock market volatility returns to the financial leverage of firms (i.e. stock price declines render firms with a high debt-to-equity ratio riskier). The second suggests that a rise in expected volatility causes current stock prices to drop so that investors can be compensated for the extra risk involved. The third focuses on stereotypes and rules of thumb or short-cuts used by people to make judgements when are busy or under time pressure (for example, they expect higher returns with lower risk from stocks of financially stable firms or they link, without any high-level reasoning, benefits with something "positive" and risks with something "negative")[1].

[1] For further details see Badshah (2013), Shefrin (2008), and Finucane et al. (2000).

From an empirical perspective, the fundamental difference between the three competing hypotheses lies in their respective implications about causality. The leverage hypothesis implies that changes in stock returns lead changes in volatility; the time-varying risk premium hypothesis implies exactly the opposite causal order; the representativeness and affect heuristics hypothesis predicts a contemporaneous than a lead-lag relationship between stock market and volatility indices.

The presence and the direction of causality between the two variables is important for investors aiming to profit from the stock and the volatility derivatives markets (Chiang, 2012). Earlier empirical investigations on the topic relied on a variety of approaches ranging from simple correlation and regression models to multivariate GARCH ones (e.g. Chiang, 2012; Hibbert et al., 2008; Bollerslev et al., 2006; Giot, 2005). Their results have been often conflicting depending on the time period considered, the statistic of volatility employed (realized or implied), and the analytical tools adopted.

This work revisits the contemporaneous and the lead-lag relations between stock market and volatility indices using notions and tools from Energy Statistics (E-statistics) (Szekely et al., 2007). Through them one may obtain a scale-invariant measure of general (linear and non linear) co-movement which, as shown by Creamer and Creamer (2016), may provide richer insights about the linkages among stochastic processes relative to alternatives. In what follows section 2 presents the analytical framework and section 3 the data, the empirical models and the results. Section 4 offers conclusions.

2. Analytical Framework

Let $X_i\ (i=1,2)$ be two random processes with characteristic functions f_i and joint characteristic function f_{12}. The *distance covariance*, $v(X_1, X_2)$, is the square root of $v^2(X_1,X_2)=\| f_{12}(s,t)-f_1(s)f_2(t)\|^2$ (where $\| \ \|$ is the norm and s and t are vectors) and measures the Brownian distance between f_{12} and $f_1 f_2$. Likewise, the *distance variance*, $v(X_i)$, is the square root of $v^2(X_i)=\| f_{ii}(s,t)-f_i(s)f_i(t)\|^2$. Once the distance covariance and variance are defined, the *Brownian distance correlation* $R(X_1,X_2)$ can be derived as

$$
(1)\quad R^2(X_1,X_2)=\begin{cases} \dfrac{v^2(X_1,X_2)}{\sqrt{v^2(X_1)v^2(X_2)}}, & v^2(X_1)v^2(X_2)>0 \\ 0, & v^2(X_1)v^2(X_2)=0 \end{cases}
$$

(Szekely and Rizzo 2013; Szekely *et al.*, 2007).

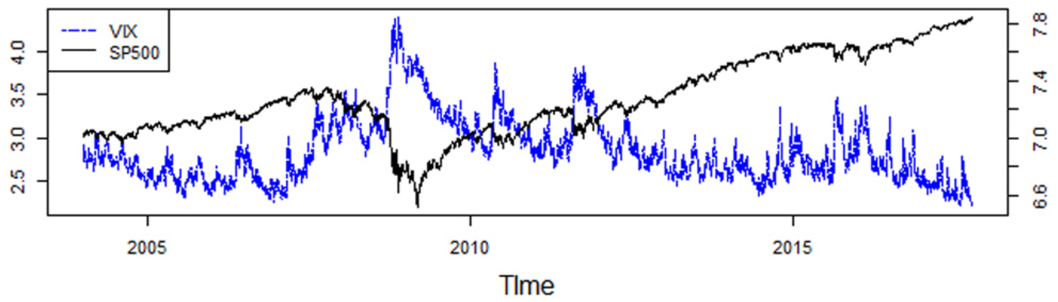
From the very definition of the norm, $v(X_1,X_2)\geq 0$ and $v(X_1,X_2)=0$ *iff* the random processes X_1 and X_2 are independent. R is an unsigned correlation coefficient taking the value of 0 under independence and the value of 1 under perfect co-movement.

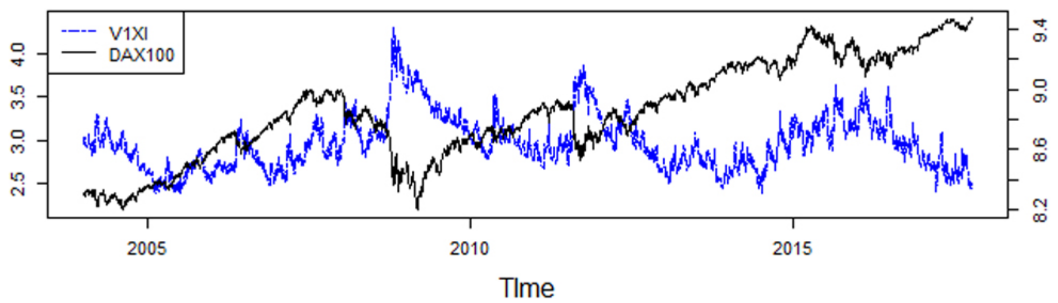
Provided that X_1 and X_2 consist of time series observations, the Brownian distance correlation may be used to investigate general co-movement of the current value of

X_i (X_{it}) on the *l*-lagged value of X_j (X_{jt-l}) $(j=1,2$ and $j \neq i)$. In particular, if $R(X_{it}, X_{jt-l}) > 0$ and l>0, then X_{jt-l} leads X_{it}. In addition, if $R(X_{it}, X_{jt-l}) > 0$ and $R(X_{it-l}, X_{jt}) = 0$, then there is a uni-directional relationship from X_{jt-l} to X_{it}. However, if $R(X_{it}, X_{jt-l}) > 0$ and $R(X_{it-l}, X_{jt}) > 0$, there is a feedback relationship between the two processes. In contrast, if $R(X_{it}, X_{jt-l}) = 0$ and $R(X_{it-l}, X_{jt}) = 0$, there is no lead-lag relationship between X_1 and X_2 (Creamer and Creamer, 2016).

3. The Data, the Empirical Models, and the Results.

The data for the empirical analysis are daily observations from three pairs of stock market and implied volatility ("fear gauge") indices, namely, the S&P 500 and the VIX, the DAX 100 and the V1XI, and the N225 and the JNIV. They have obtained from the CBOE and the investing.com websites and they refer to the period 2/1/2004 to 6/10/2017 (a total of 3593 observations). As known "fear gauge" indices, are derived from stock options and represent a consensus forecast over the expected short-run (typically 30 calendar days) stock market volatility (e.g. Chiang, 2012; Whaley, 1993). Figure 1 presents the natural logarithms of the six time series. It is evident that, on most occasions, the stock and the implied volatility indices for a given market move in opposite directions.





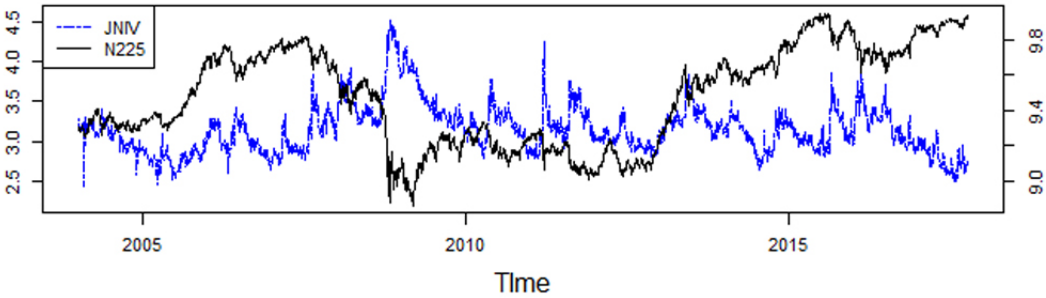


Figure 1. Logarithmic stock and implied volatility indices

Earlier empirical works (e.g. Giot, 2005) suggested that the strength and the pattern of the relationship between the stock market and the volatility indices may depend on volatility levels. Here, to allow for such possibility we have applied the multiple breakpoint test of Bai and Perron (2003) to the three log "fear gauge" indices and we have estimated the Brownian distance correlation coefficients at a number of different sub-periods. Table 1 (panels (a) to (c)) presents the test results. In all cases, the test detected four break points. It is noteworthy that the two first breaks occurred at about the same time for all log implied volatility series while the third and the fourth occurred at dates up to eight months apart. Also, the time periods between the first and the third break (which include the financial crisis of 2008/9 and the nervous years that followed) are characterized by higher implied volatility relative to the rest.

Table 1: Results of the Bai-Perron on the Log Implied Volatility Series

(a) VIX**

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	1313.567*	8.58	2/7/2007	2.6*
1 vs 2	2350.181*	10.13	7/9/2009	3.4*
2 vs 3	198.446*	11.14	26/1/2012	3.13*
3 vs 4	72.419*	11.83	21/1/2014	2.75*
4 vs 5	0	11.25		2.66*

**, Maximum no breaks: 5; trimming: 0.15; level of significance: 0.05(assessed using HAC standard errors)

(b) V1XI **

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	1065.662*	8.58	18/7/2007	2.76*
1 vs 2	720.402*	10.13	9/11/2009	3.21*
2 vs 3	582.111*	11.14	6/9/2012	3.01*
3 vs 4	127.117*	11.83	29/9/2014	2.73*
4 vs 5	0	12.25		3*

**, Maximum no of breaks: 5; trimming: 0.15; level of significance: 0.05 (assessed using HAC standard errors)

(c) JNIV **

Null Hypothesis: *L*+1 vs *L* breaks	Scaled F-statistic	Critical Value	Break Dates	Average over the time interval
1 vs 0	898.871*	8.58	10/8/2007	2.96*
1 vs 2	817.738*	10.13	21/8/2009	3.57*
2 vs 3	182.723*	11.14	2/12/2011	3.22*
3 vs 4	24.075*	11.83	1/5/2014	3.16*
4 vs 5	0	12.25		3.05*

**, Maximum no of breaks: 5; trimming: 0.15; level of significance: 0.05 (assessed using HAC standard errors)

Prior to the estimations we have evaluated the stationarity of all time series using the ADF test. The log stock indices turned out to be non stationary for the total period and for all sub-periods. The log implied volatility indices turned out to be stationary in a number of sub-periods. All first log differences (returns), however, are stationary. To avoid mixing non stationary and stationary time series we have conducted the empirical analysis on returns.

Following Creamer and Creamer (2016), we have estimated Brownian distance correlations at l=1,2, …, 7 lags. Table 2 (panels (a) to (c)) presents the results . Starting with the pair (S&P 500, VIX), the Brownian distance correlations between current implied volatility returns and stock market returns at the different lags are all statistically significant for the total period and for the fifth sub-period; there is also a large number of statistically significant correlations in the remaining sub-periods, especially at $1 \leq l \leq 4$. The Brownian distance correlations between current stock market returns and implied volatility returns at the different lags are all statistically significant for the total period; there is also a relatively small number of statistically significant correlations in the remaining sub-periods, primarily at l=1. On the basis of the values and the statistical significance of the estimated distance correlations one may conclude that, although causality between the S&P 500 and the VIX may be bi-directional, the influence of lagged S&P 500 returns on current VIX returns has been far more stronger than that of lagged VIX returns on current S&P 500 returns. Therefore, between the leverage and the volatility feedback hypothesis the data appear to provide more support to the former.

Table 2: Brownian Distance Correlations

(a) S&P 500 and VIX returns

	Number of lags							
Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
	Null hypothesis: SP500 does not lead VIX							
Total	0.207**	0.481**	0.086**	0.105**	0.076**	0.055**	0.059**	0.064**
Sub-period 1[+]	0.103**	0.561**	0.074	0.155**	0.094*	0.075	0.059	0.083
Sub-period 2	0.151**	0.597**	0.131**	0.102	0.111*	0.098	0.077	0.093
Sub-period 3	0.304**	0.434**	0.115*	0.133**	0.136**	0.070	0.099	0.076
Sub-period 4	0.296**	0.354**	0.085	0.109	0.106	0.081	0.097	0.077
Sub-period 5	0.282**	0.483**	0.115**	0.116**	0.094**	0.094*	0.092*	0.089*
	Null hypothesis: VIX does not lead SP500							
Total	0.207**	0.066**	0.067**	0.062**	0.062**	0.050*	0.051**	0.062***
Sub-period 1	0.103**	0.074	0.067	0.064	0.061	0.067	0.057	0.077
Sub-period 2	0.151**	0.124*	0.091	0.092	0.091	0.077	0.093	0.103
Sub-period 3	0.304**	0.102*	0.095	0.088	0.092	0.094	0.095	0.077
Sub-period 4	0.296**	0.113*	0.084	0.076	0.070	0.076	0.076	0.092
Sub-period 5	0.282**	0.103**	0.097*	0.097**	0.110**	0.080	0.065	0.086*

[+], the sub-periods 1 to 5 are: 3/1/2004 to 2/7/2007, 3/7/2007 to 7/9/2009, 8/9/2009 to 26/1/2012, 27/1/2012 to 21/1/2014, and 22/1/2014 to 6/10/2017, respectively; *, p≤0.05, **, p≤0.01

(b) DAX 100 and V1XI

Period	Number of lags 0	1	2	3	4	5	6	7
Null hypothesis: DAX 100 does not lead V1XI								
Total	0.772**	0.077**	0.083**	0.076**	0.072**	0.071**	0.048*	0.044*
Sub-period 1+	0.785**	0.132**	0.108**	0.066	0.091*	0.077	0.056	0.047
Sub-period 2	0.773**	0.109*	0.091	0.120*	0.104	0.113*	0.072	0.112*
Sub-period 3	0.818**	0.092	0.131**	0.117**	0.111**	0.110**	0.120**	0.067
Sub-period 4	0.799**	0.105	0.074	0.088	0.137**	0.087	0.091	0.097
Sub-period 5	0.725**	0.096*	0.105**	0.092*	0.080	0.075	0.066	0.063
Null hypothesis: V1XI does not lead DAX 100								
Total	0.772**	0.056**	0.070**	0.055**	0.047*	0.055**	0.047*	0.043*
Sub-period 1	0.785**	0.057	0.077	0.080	0.048	0.085	0.080	0.060
Sub-period 2	0.773**	0.073	0.107*	0.085	0.101	0.079	0.113*	0.088
Sub-period 3	0.818**	0.110**	0.119**	0.099*	0.084	0.065	0.072	0.083
Sub-period 4	0.799**	0.096	0.085	0.074	0.083	0.072	0.102	0.086
Sub-period 5	0.725**	0.074	0.097*	0.078	0.094*	0.093*	0.055	0.065

+, the sub-periods 1 to 5 are: 3/1/2004 to 18/7/2007, 19/7/2007 to 9/11/2009, 10/11/2009 to 6/9/2012, 7/9/2012 to 29/9/2014, and 30/9/2014 to 6/10/2017, respectively; *, p≤0.05, **, p≤0.01

(c) N225 and JNIV

Period	Number of lags 0	1	2	3	4	5	6	7
Null hypothesis: N225 does not lead NIV								
Total	0.554**	0.113**	0.082**	0.077**	0.060**	0.061**	0.052**	0.055**
Sub-period 1+	0.420**	0.116**	0.092*	0.098**	0.071	0.088*	0.062	0.074
Sub-period 2	0.706**	0.137**	0.080	0.086	0.109	0.081	0.091	0.086
Sub-period 3	0.719**	0.124**	0.139**	0.113*	0.093	0.125*	0.081	0.119*
Sub-period 4	0.346**	0.128**	0.080	0.084	0.077	0.090	0.072	0.073
Sub-period 5	0.589**	0.125**	0.117**	0.107**	0.080	0.105**	0.083	0.092*
Null hypothesis: JNIV does not lead N225								
Total	0.554**	0.059**	0.058**	0.054**	0.056**	0.049*	0.049*	0.038
Sub-period 1	0.420**	0.064	0.062	0.070	0.062	0.085	0.068	0.086*
Sub-period 2	0.706**	0.092	0.078	0.110	0.095	0.086	0.088	0.070
Sub-period 3	0.719**	0.114*	0.116*	0.072	0.102	0.087	0.075	0.066
Sub-period 4	0.346**	0.093	0.091	0.102*	0.067	0.076	0.072	0.072
Sub-period 5	0.589**	0.109**	0.087	0.093*	0.086	0.078	0.088*	0.094*

+, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; *, p≤0.05, **, p≤0.01

The contemporaneous Brownian distance correlation is considerably lower than that between current VIX returns and the lagged (by one) S&P 500 returns in all periods considered providing, thus, more evidence in favour of the leverage relative to representativeness and affect heuristics hypothesis. Finally, no clear pattern appears to exist between the log implied volatility level in a given sub-period and the values of the respective distance correlations at the various lags.

For the pair (DAX 100, V1XI), the distance correlations involving return series with strictly positive lags, point to bi-directional causality (especially for the total, the third, and the fifth sub-period). The contemporaneous distance correlation is very high relative to those involving one lag in the DAX 100 or in the V1XI returns. The data, therefore, provide very strong support to the representativeness and affect heuristics hypothesis relative to the competing ones. The estimations results for the pair (N225, JNIV) are quality-wise similar to those for the pair S&P 500 and VIX with regard to leverage vs volatility feedback hypothesis; they, however, favour the representativeness and affect heuristics relative to the leverage hypothesis.

As noted in the Introduction, the measure (Brownian distance correlation) obtained through the E-statistics is general, in the sense that it captures both linear and non linear co-movement. Standard measures of association such as the Person correlation coefficient and standard tests of causality such as the Granger one assume that the underlying relationships are linear. It would be certainly interesting to investigate whether the linear and the more general approaches to co-movement and causality lead to similar results.

Table 3 presents the values of the Pearson correlation coefficient for contemporaneous changes in the stock and the “fear gauge” indices. The results are consistent with what is reported in Table 2 (first column); higher, in absolute value terms, Pearson correlation coefficients are associated with higher Brownian distance correlation coefficients. Moreover, both measures suggest that the strongest contemporaneous association is the one between the DAX and the V1XI and the weakest between the SP500 and the VIX.

Table 3: Pearson Correlation Coefficients

Period	SP500 and VIX	DAX and V1XI	N225 and JNIV
Total	-0.193**	-0.736**	-0.561**
Sub-period 1+	-0.101**	-0.829**	-0.348**
Sub-period 2	-0.084**	-0.637**	-0.688**
Sub-period 3	-0.322**	-0.827**	-0.761**
Sub-period 4	-0.318 **	-0.829**	-0.422**
Sub-period 5	-0.319**	-0.747**	-0.634**

+, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; **, p≤0.01

Table 4 present the results of the linear Granger causality tests. The null hypothesis that changes in the SP500 do not lead changes in the VIX is strongly rejected. The null, hypothesis, however, that changes in the VIX do not lead changes in the SP500 is consistent with the real world data in all but one sub-periods. The Granger test, therefore, points to uni-directional causality whereas the Brownian correlation coefficient (Table 2(a)) has largely pointed to a bi-directional one. The null hypothesis that changes in the DAX do not lead changes in the V1XI is rejected for three sub-periods (but not for the total period). There is no period, however, in which changes in the V1XI lead those in the DAX. Here, again, the Granger test offers some evidence of uni-directional causality whereas the Brownian motion correlation coefficient has pointed to a largely bi-directional one (Table 2 (b)). Very similar are the results of the Granger test for the pair N225 and JNIV.

Another notable difference between the results in Tables 2 and 4 is that whereas the non linear measure detects quite a few statistically significant associations at 3, 4, and (in certain cases) even at 7 lags, the Granger test indicates that the effect of shocks is very short-lived (the optimal lag length is everywhere less than or equal to 2). The fact that the Brownian correlation coefficient suggests that the effect of shocks has potentially a considerable duration whereas the Granger test indicates that the effect of shocks dies out very quickly must be attributed to the assumptions underlying the two approaches. The standard Granger test captures linear relations only; the Brownian correlation coefficient, however, works equally well with linear and non linear linkages (Creamer and Creamer, 2016).

Table 4: Granger Causality Tests (F Values)

	Null hypothesis:		
Period	**SP500 does not lead VIX**	**DAX 100 does not lead V1XI**	**N225 does not lead JNIV**
Total	683.136** (2)	1.604 (1)	0.142 (1)
Sub-period 1⁺	362.527** (2)	3.937* (1)	6.013* (1)
Sub-period 2	366.541** (1)	8.626** (1)	0.366 (1)
Sub-period 3	220.076** (1)	1.618 (1)	2.116 (1)
Sub-period 4	87.226** (2)	0.007 (1)	3.765 (1)
Sub-period 5	206.044** (2)	4.848* (1)	4.718* (1)
Null hypothesis:			
	VIX does not lead SP500	**V1XI does not lead DAX100**	**NJIV does not lead N225**
Total	2.028 (2)	0.006 (1)	0.027 (1)
Sub-period 1⁺	1.535 (2)	1.260 (1)	0.001 (1)
Sub-period 2	13.45** (1)	0.006 (1)	0.388 (1)
Sub-period 3	0.863 (1)	0.228 (1)	13.963** (1)
Sub-period 4	1.763 (2)	0.935 (1)	2.870 (1)
Sub-period 5	1.651 (2)	0.441 (1)	2.340 (1)

⁺, the sub-periods 1 to 5 are: 3/1/2004 to 10/8/2007,11/8/2007 to 21/8/2009, 22/8/2009 to 2/12/2011, 3/12/2011 to 1/5/2014, and 2/5/2014 to 6/10/2017; *, $p \leq 0.05$, **, $p \leq 0.01$; optimal number of lags in parentheses, determined using the Bayesian Information Criterion (BIC).

4. Conclusions

In this study we have employed the Brownian distance correlation coefficient to investigate empirically the validity of three competing hypotheses (leverage, time-varying risk premium, and representativeness and affect heuristics) with regard to the contemporaneous and the lag-lead linkages between stock market and implied volatility indices. For the empirical analysis we have utilized daily observations over 2004 to 2017 from the S&P 500, the DAX 100, the N225, the VIX, the D1XI, and the JNIV.

The empirical results appear to provide strong support to the leverage relative to the volatility feedback hypothesis for the pairs (S&P 500, VIX), and (N225, JNIV). This is in line with the findings of Bollerslev et al. (2006). For the pair (DAX 100, V1XI), and in accordance with what has been reported by Chiang (2012), the evidence points to a bi-directional causality.

The contemporaneous Brownian correlations between stock market and volatility returns have received much higher values relative to those involving lags for the pairs (DAX 100, V1XI) and (N225, JNIV). This is consistent with the findings of Badshah (2013) and Hibbert et al. (2008) and favours the representativeness and affect heuristics hypothesis relative to the leverage and the time-varying risk premium hypotheses. For the pair (S&P 500, VIX), the evidence favours a lead-lag relation over a contemporaneous one.

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