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INVESTIGATION OF ASYMMETRIC DYNAMICS OF BORSA ISTANBUL INDEX WITH QUANTILE UNIT ROOT TEST

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Abstract

The main contribution of the study is the empirical examination of the Borsa Istanbul Index using Koenker and Xiao's (2004) quantile unit root test, which provides robust inferences for non-normal processes based on the quantile autoregression approach. The study contributes to the portfolio formation based on quantile regression for future studies and highlights the importance of understanding asymmetric inferences in shock magnitude and sign for asset pricing and forecasting in the securities market. The findings indicate that the dynamic structure of the index displays asymmetrical behaviour, introducing quantile perspectives to index dynamics in contrast to conventional unit root methodologies based on the least squares regression method.

Keywords: Quantile autoregression, nonparametric test, asymmetry, stock exchange, asset pricing

1. Introduction

The Efficient Market Hypothesis (EMH), introduced by Fama (1970), asserts that according to classical finance theory, information is rapidly incorporated into all asset prices, preventing participants from achieving returns surpassing the market return. However, over the last fifty years, various anomalies have been identified, including weekend, end-of-day, herd psychology, and trend effects, challenging the foundations of the EMH. Psychologists and experimental economists have argued that behavioural biases such as overconfidence, overreaction, and regret play a role in human decision-making under uncertainties. In their study, Grossman and Stiglitz (1980) criticize the efficient market hypothesis, contending that truly efficient markets, where information is perfectly known by everyone, do not exist. Grossman (1976) and Grossman and Stiglitz (1980) suggest that if markets were genuinely efficient, there would be no profitable investments resulting from information asymmetry among investors. The literature reflects a lack of consensus between advocates of information efficiency and behavioural finance, with ongoing theoretical and empirical studies. As emphasized by Bernstein (1999), the market equilibrium central to the EMH rarely occurs in practice, and market efficiency is better defined by evolutionary processes. Given the increased access to information in recent years, there is a growing importance in studies focusing on theoretical and empirical models related to information supply and information demand in information flow.

Shocks resulting from news flow to the markets and their persistence play a crucial role in financial asset forecasting models. In the finance literature, the unit root hypothesis is considered the primary method for assessing the permanence of shocks on financial variables. This hypothesis relies on an autoregressive process designed for optimal performance under the assumption of normality. However, given that variables in financial markets often exhibit a heavy-tailed (leptokurtic) distribution, it is crucial to employ estimation and inference procedures that produce robust results against deviations from the normality assumption. In classical regression, one assumption for the least squares estimators to be effective is that the series follows a normal distribution. Recognizing the need for robust estimators under deviations from normality, quantile unit root tests robust to such deviations, based on quantile autoregression, have been introduced to the literature. These tests are designed to exhibit strong power across various error distributions. Koenker and Xiao (2004) propose new tests for the unit root hypothesis based on the quantile autoregression (QAR) approach in a univariate context. Unlike standard unit root tests applied to examine the Efficient Market Hypothesis (EMH), which generally focuses on the average behaviour of stock prices, these quantile-based tests consider the impact of shock magnitudes and signs on the index.

This study proposes a quantile autoregression approach to assess market efficiency, establishing a connection between the types of news (good and bad) entering the market and their quantiles. By modelling stock market returns across various quantile levels, on the magnitude and sign of the shocks, the study reveals different market conditions. The aim is to investigate the persistence of shocks in the same series at different frequencies (daily, weekly, monthly, quarterly, annual) and explore the series' asymmetric dynamic structure. Consequently, instead of treating the series as a whole, it undergoes examination through classification based on shock magnitude and sign. This study represents the first to provide robust quantile autoregression evidence for the efficiency of the Borsa Istanbul stock index. The empirical analysis delves into market activity at different frequencies over the long term. Moreover, the study is poised to contribute to future research on optimal portfolio creation, leveraging both linear and non-linear quantile autoregressive processes. Additionally, the exploration of asymmetric dynamics holds significant implications for asset pricing models.

The persistence of good or bad news in the market holds significant importance for predicting price movements in stock markets. Quantile unit root tests, grounded in the quantile autoregression process, play a crucial role in forecasting price movements. These tests enable the examination of shock magnitudes and asymmetry caused by good and bad news separately, offering a nuanced perspective beyond treating the series as a whole. Methods for detecting the presence of a unit root in semi-parametric time series models are subject to ongoing theoretical and empirical exploration. One approach to enhance power performance involves the utilization of robust estimators. The literature provides a theoretical foundation encompassing methods robust to deviations from assumptions. Notable among these are studies on M estimation and inferences (Cox and Llatas, 1991; Knight, 1991; Phillips, 1995; Lucas, 1995; Rothenberg and Stock, 1997; Juhl, 1999; Xiao, 2001). The theoretical discourse on the quantile regression method and subsequent robust estimators initiated by Koenker and Bassett (1978) continues, with contributions from scholars such as Weiss (1987), Knight (1989), Koul and Saleh (1995), Koul and Mukherje (1994), Hercé (1996), Hallin and Jureckova (1999), and Rogers (2001).

Tests based on quantile autoregression provide valuable insights into the dynamics and persistence of financial time series. There are t-ratio, Kolmogorov-Smirnov, and Cramer von Mises type tests, relying on the estimation of selected quantiles within specific series intervals. OLS regression estimates lose their effective predictive properties when deviations from the normality assumption occur. In such instances, estimators based on quantile autoregression emerge as robust alternatives. However, when the normality assumption holds, the application of quantile regression results in a loss of efficiency. Furthermore, quantile unit root tests demonstrate enhanced power in the presence of asymmetric dynamics compared to classical unit root tests.

In the finance literature, various models explain empirical momentum and reversal phenomena, attributing them to stock prices under- or overreacting to good or bad news. Barberis, Shleifer, Vishny (1998) propose an investor sensitivity model where they assume returns to be a random walk, but investors are unaware of this, leading to poor stock price reactions to earnings announcements. Baur et. al. (2012) examine positive dependency (negative return) in lower quantiles and negative dependence (positive return) in upper quantiles. Engle and Manganelli (2004) introduce Conditional autoregressive value at risk (CAViaR), a VaR model based on quantile regression. Feng, Chen, Bassett

(2008) and Ma, Pohlman (2008) introduce quantile momentum measurements for creating momentum portfolios in asset management. Quantiles are increasingly utilized in optimal portfolio selection, as demonstrated by Chambers (2009), Bhattacharya (2009), Giovannetti (2013), and de Castro and Galvao (2019). Bassett and Chen (2002) examine the quantile regression method as an addition to the style classification toolkit, enhancing portfolio style classification by identifying the impact of style on the conditional return distribution beyond the expected value. Ma & Pohlman (2008) address issues in equity return forecasting and portfolio construction, introducing quantile regression methods to improve forecasting and portfolio outcomes. Quantile portfolio models aimed at modelling economic behaviour have been used by Chambers (2007), Bhattacharya (2009), Giovannetti (2013), and de Castro and Galvao (2019). Literature on optimal portfolio allocation includes studies by Kulldorff (1993), Föllmer and Leukert (1999), He and Zhou (2011), and Brown and Sim (2009). Castro et. al.'s (2022) study introduces a model for optimal portfolio allocation that maximizes the **τ**-quantile of portfolio return for **τ** ∈ (0, 1), addressing the preferences of investors with quantitative inclinations. The increasing importance of the quantile approach in portfolio selection is evident in the literature.

In Bahmani et. al. (2016) study, weekly stock prices data from eight countries with transition economies (Bulgaria, Croatia, Czech Republic, Hungary, Lithuania, Poland, Romania, and Russia) during the period 2000–2015 are utilized. The weak form of the market hypothesis is tested using the quantile unit root test, revealing that stock markets are weak-form efficient for most countries, except Bulgaria, Romania, and Russia. Novak (2019) employs the quantile autoregression approach to assess the market efficiency of the Croatian stock market, analysing daily CROBEX returns from 2000 to 2019. The study rejects the basic hypothesis when examining the weak form of market efficiency with quantile unit root tests. The observed ineffective predictable behaviour of CROBEX suggests the potential for investors to achieve abnormal profits. Jiang and Li's (2020) study introduces a new measure of market efficiency to analyse efficiency dynamics across various quantile levels in Chinese, Japanese, and US stock markets. The findings indicate that Japanese and US stock markets exhibit efficiency under normal conditions (mid quantiles) rather than during bull market (high quantiles) or bear market (low quantiles) conditions. In contrast, the Chinese stock market is deemed inefficient across all quantiles, with the US stock markets showing smaller deviations from efficiency in most periods. Nartea (2021) examines the stationarity of daily real stock prices in 12 Asia-Pacific countries (Australia, China, Hong Kong, Indonesia, Japan, Malaysia, New Zealand, Philippines, Singapore, South Korea, Taiwan, and Thailand) from 1991 to 2020. The results suggest overall stability in stock prices in higher quantiles. Furthermore, there is evidence of asymmetry in stock price dynamic adjustments in the upper deciles, where larger shocks are associated with faster mean reversion, and conversely, smaller shocks are linked to nonstationarity. There are also studies in the literature that measure the degree of asymmetry in the return-volatility relationship with quantile regression (Agbeyegbe, 2015; Badshah, 2013; Badshah et. al., 2016; Bekiros et. al., 2017).

In the financial and economic literature, the persistence of shocks is typically characterized by the unit root hypothesis. Traditional unit root tests for Borsa Istanbul have been commonly employed in studies (Özdemir, 2022). The literature indicates that standard unit root tests primarily focus on the average behaviour, neglecting the magnitude and signs of shocks. These tests assume a constant rate at which stock prices adjust toward equilibrium, irrespective of the shock's size or sign. Consequently, when the assumptions of traditional unit root tests are not met in financial markets, the rejection of the unit root fundamental hypothesis tends to be limited. Studies in the literature underscore that information efficiency is lower in emerging markets. This emphasizes that, especially in emerging markets like Turkey, where low-frequency information collection and processing costs are higher, the lower information efficiency leads to a more extended period for information to be fully reflected in asset prices.

The remainder of this study are structured as follows: Chapter 2 introduces the QAR model along with new tests and robust inferences based on QAR. Section 3 presents the data, while Section 4 assesses the results of the empirical analysis. Finally, Section 5 offers concluding remarks.

2. Methodology

Considering the heavy-tail behaviour often observed in financial time series in various empirical studies, it becomes crucial to utilize estimation and inference procedures that are robust to deviations from Gaussian conditions in non-stationary time series. The quantile regression approach becomes particularly relevant in this context, as it allows researchers to explore a range of conditional quantitative functions rather than focusing solely on a conditional measure of central tendency. Quantile autoregression methods offer a robust framework for inference, enabling the investigation of various forms of conditional heterogeneity by exploring different conditional quantiles. The quantile unit root test proposed by Koenker and Xiao (2004) introduces new tests based on quantile autoregression. These tests evaluate statistics on selected quantiles or over a specific range of quantiles, utilizing estimations based on t-ratio tests, Kolmogorov-Smirnov, or Cramer-Von Mises type tests. Notably, these new tests provide robust results even in the absence of normality assumptions, addressing a broader set compared to existing methods in the literature. While the quantile unit root test demonstrates good power under non-normal conditions, its effectiveness diminishes when applied under normality conditions. Furthermore, quantile unit root tests facilitate the examination of asymmetric dynamics and exhibit superior power compared to classical unit root tests (Koenker-Xiao, 2006). Before delving into the quantile autoregressive process, it is essential to consider the ADF (Dickey-Fuller, 1979) regression model, an extension of the first-order autoregression model of the unit root process.

$$y_t = \alpha_1 y_{t-1} + \sum_{j=1}^{q} \alpha_{j+1} \Delta y_{t-j} + u_t \tag{1}$$

The autoregressive coefficient α_1 in the equation above plays a crucial role in assessing persistence, serving as an indicator for the presence of a unit root in financial time series. Specifically, if α_1 equals 1, the series contains a unit root, signifying persistence in the process. Conversely, If $|a_1| < 1$ situation occurs, the process is stationary. Introducing the σ- region produced by $\{u_s, s £ t\}$ via F_t , the conditional quantile t. of y_t on F_{t-1} is defined as follows:

$$Q_{y_t}(\tau / \mathcal{F}_{t-1}) = Q_u(\tau) + \alpha_1 y_{t-1} + \sum_{j=1}^{q} \alpha_{j+1} \Delta y_{t-j} \tag{2}$$

In the above equation, for j=1,…,q when it is defined as

$$Q_u(\tau) = \alpha_0(\tau), \alpha_j = \alpha_j(\tau) \tag{3}$$

$$\alpha(\tau) = (\alpha_0(\tau), \alpha_1(\tau), \dots, \alpha_{q+1}(\tau))', x_t = (1, y_{t-1}, \Delta y_{t-1}, \dots, \Delta y_{t-q})', \tag{4}$$

we obtain the following equation:

$$Q_{y_t}(\tau\,/\,\mathcal{F}_{t-1}) = x_t'\alpha(\tau) \tag{5}$$

Here, the estimation of the linear quantile autoregression model includes a solution to the following minimization problem.

$$\min_{\alpha\,\epsilon\,\mathbb{R}^2} \sum_{t=1}^{n} \rho_\tau(y_t - x_t^T b) \tag{6}$$

Here, the ρ_τ function is the piecewise control function shown as $\rho_\tau(u) = u(\tau - I(u < 0))$ proposed by Koenker and Bassett (1978). With $0 < \tau < 1$, the I function as an indicator function is as follows:

$$\rho_\tau(u) = \begin{cases} \tau|u|, & u \geq 0 \\ (1-\tau)|u|, & u < 0 \end{cases} \tag{7}$$

The τ quantile with $0<\tau<1$ is defined as the solution to the minimization problem:

$$\min_{b\,\in\,\mathbb{R}^K} \left[\sum_{t\,\in\{t:y_t \geq x_t b\}} \tau|y_t - x_t b| + \sum_{t\,\in\{t:y_t < x_t b\}} (1-\tau)|y_t - x_t b| \right] \tag{8}$$

The minimization problem yields the solution attributed to $\hat{\alpha}(\tau)$, representing the τ. quantile autoregression process viewed as a function of τ. The estimation of the conditional density function of y_t is accomplished through the difference quotients for selected quantiles of τ.

$$\hat{f}_{y_t}(\tau/x_t) = (\tau_i - \tau_{i-1})/(\,\hat{Q}_{y_t}(\tau_i/x_t) - \hat{Q}_{y_t}(\tau_{i-1}/x_t)) \tag{9}$$

The approach based on the quantile autoregression process offers a more robust method for testing the unit root hypothesis compared to traditional unit root tests relying on least squares. Koenker-Xiao's (2004) t-ratio test statistic is defined as:

$$t_n(\tau) = \frac{f\big(\widehat{F^{-1}(\tau)}\big)}{\sqrt{\tau(1-\tau)}} (Y_{-1}^T P_X Y_{-1})^{1/2} (\widehat{\alpha_1}(\tau) - 1). \tag{10}$$

$f\big(\widehat{F^{-1}(\tau)}\big)$ is the consistent estimator of $f\big(F^{-1}(\tau)\big)$. Y_{-1} is a vector consisting of lagged values of the dependent variable (y_{t-1}) and P_X is the projection matrix onto the space orthogonal to $X =$

$(1, \Delta y_{t-1}, \dots, \Delta y_{t-q})$. The sparsity function s(τ) is defined in two ways: (1) inverse of the density function or (2) derivative of the quantile function:

$$s(\tau) = F^{-1\prime}(\tau) = 1/f\left(F^{-1}(\tau)\right) \tag{11}$$

Here is relevant literature on $f\left(F^{-1}(\tau)\right)$ estimation, including the studies of Siddiqui (1960) and Bofinger (1975):

$$f_n\left(F_n^{-1}(\tau)\right) = \frac{2h_n}{F_n^{-1}(\tau + h_n) - F_n^{-1}(\tau - h_n)} \tag{12}$$

$F_n^{-1}(.)$ is an estimator approximation of $F^{-1}(.)$ where h_n is a bandwidth that approaches 0 as n→∞. The bandwidth used in this study is the Bofinger (1975) bandwidth as commonly adopted in the literature:

$$h_B = n^{-1/5}\left[\frac{4.5\phi^4(\Phi^{-1}(\tau))}{[2(\Phi^{-1}(\tau))^2 + 1]^2}\right]^{1/5} \tag{13}$$

Here, the functions $\phi(.)$ and $\Phi(.)$ represent the density and cumulative distribution functions of the standard normal distribution, respectively.

At any chosen τ, the test statistic $t_n(\tau)$ is the quantile regression counterpart of the ADF t-test statistic based on least squares regression. Unit root tests based on quantile autoregressive processes can be formed by representative quantiles (low quantile, median, high quantile). Alternatively, the examination can cover the range of selected quantiles with $\tau \epsilon \mathcal{T}$. Another approach is to test over a range of quantiles rather than just focusing on selected ones. The Kolmogorov-Smirnov (KS) test based on the quantile regression process for τ ∈ T is as follows:

$$QKS_t = \sup_{\tau \epsilon \mathcal{T}} |t_n(\tau)| \tag{14}$$

with $\tau_0 > 0$, $\tau \epsilon \mathcal{T} = [\tau_0, 1 - \tau_0]$.

In applications, $t_n(\tau)$ can be calculated with $\{\tau_i = i/n\}_{i=1}^{n}$. Thus, the QKS_t statistics can also be generated by taking its maximum on $\tau_i \epsilon \mathcal{T}$. Evaluation can be made not only for the selected quantiles (t) by comparing the calculated $t_n(t)$ test statistic with the critical values, but also by comparing the Quantile Kolmogorov-Smirnov (QKS) test and its critical value for the series in general. While the limiting distributions of both $t_n(\tau)$ and QKS tests are not standardized, Koenker and Xiao (2004) suggest using a resampling procedure (bootstrap number = 10,000 in our study) to approximate small sample distributions.

Thus, states can be examined for some quantiles, such as various decimals. The practical importance of this feature can be examined, as different quantiles correspond to shocks of different signs and magnitudes. Thus, asymmetric effects are observed when examining the persistence of shocks.

3. Data

In this study, which examines the asymmetric dynamics of the BIST100 index on a quantile basis, daily, weekly, monthly, quarterly, and annual closing indices covering the period between March 2003 and March 2023 are used. It is used by taking the natural logarithm of the index data.

Descriptive statistics are as follows:

Table 1: Descriptive statistics

	Daily	Weekly	Monthly	Quarterly	Annual
Mean	6.4654	6.468	6.4762	6.4739	6.6537
Median	6.5614	6.5622	6.5755	6.5473	6.5755
Standard dev.	0.7196	0.7192	0.7304	0.7517	0.8649
Kurtosis	0.7587	0.7674	0.8934	1.0885	0.9832
Skewness	-0.0131	-0.0066	0.0513	0.0826	0.8138
JB prob.	0.0000	0.0000	0.0000	0.0000	0.0000
Min	4.4878	4.544	4.5513	4.5513	5.2271
Max	8.6414	8.6142	8.6142	8.6142	8.6142
Obs.	5024	1042	241	81	21

When we look at the descriptive statistics in Table 1, there is a minimal increase in the mean, median and standard deviation values as we go from the daily data to the annual data. When the kurtosis and skewness are examined for the assumption of normality, it is seen that the series are not normally distributed. In addition, as seen from the JB probability values, the null hypothesis of normal distribution is rejected. The numbers of observation values are 5024, 1042, 241, 81, and 21 for daily, weekly, monthly, quarterly, and annual frequencies, respectively.

4. Findings

In this section, we present the results of the Augmented Dickey-Fuller (ADF) and Koenker-Xiao's (2004) quantile unit root tests conducted on the BIST100 index values at various frequencies. The quantile unit root test is applied across different deciles (0.1, 0.2, ..., 0.9) for the BIST100 index at daily frequencies, and critical values are determined using the Bootstrap method in Matlab.

Table 2 provides the quantile unit root test results for the daily frequency of the BIST100 index. While the ADF test suggests the presence of a unit root in the dataset, interestingly, the data is observed to be stationary at higher quantiles (specifically, [0.7, 0.8, 0.9]), leading to the rejection of the null hypothesis for daily data. This implies that the index tends to revert to the mean in response to good news, particularly at high quantiles. Conversely, a unit root process is detected in the face of medium and bad news, corresponding to medium and low quantiles in the stock market, respectively. However, when the ADF test statistics results for the data set for the period 2003-2023 are examined, we see that the series exhibits unit root in terms of the daily frequency data set.

Table 2: Koenker-Xiao (2004) Quantile Unit Root Test Results (daily)

τ (Quantiles)	Coefficient (α_1)	Results	$t_n(\tau)$	Critical Values
0.1	1.0029	1	4.5870	-2.7673
0.2	1.0027	1	5.7327	-2.8151
0.3	1.0022	1	5.5463	-2.7976
0.4	1.0012	1	3.6015	-2.7438
0.5	1.0003	1	1.0035	-2.6543
0.6	0.9997	1	-0.8421	-2.5635
0.7	0.9989	0	-3.0383**	-2.4450
0.8	0.9980	0	-5.0526**	-2.3800
0.9	0.9971	0	-4.6971**	-2.1245
QKS		0	5.7327**	2.7365
ADF		1	-0.0846	-2.8619

*Note: ** indicate 5% significance level.*

Table 3: Koenker-Xiao (2004) Quantile Unit Root Results (weekly)

τ (Quantiles)	Coefficient (α_1)	Results	$t_n(\tau)$	Critical Values
0.1	1.0039	1	1.0764	-2.5720
0.2	1.0027	1	1.0090	-2.6507
0.3	1.0004	1	0.1815	-2.6987
0.4	0.9993	1	-0.3892	-2.6338
0.5	0.9999	1	-0.0507	-2.5974
0.6	0.9984	1	-0.9339	-2.5746
0.7	0.9980	1	-1.0821	-2.5322
0.8	0.9986	1	-0.7240	-2.4507
0.9	0.9960	1	-1.4525	-2.5116
QKS		1	1.4525	2.7834
ADF		1	-0.1257	-2.8641

*Note: * indicate 5% significance level.*

Table 4: Koenker-Xiao (2004) Quantile Unit Root Results (monthly)

τ (Quantiles)	Coefficient (α_1)	Results	$t_n(\tau)$	Critical Values
0.1	0.9930	1	-0.5402	-2.6102
0.2	1.0121	1	1.1838	-2.4517
0.3	1.0000	1	0.0000	-2.5292
0.4	0.9965	1	-0.3335	-2.5692
0.5	0.9835	1	-1.6278	-2.6105
0.6	0.9880	1	-1.3749	-2.5390
0.7	0.9922	1	-0.9167	-2.5587
0.8	0.9929	1	-0.8101	-2.5772
0.9	0.9949	1	-0.2970	-2.5309
QKS		1	1.6278	2.8144
ADF		1	-0.4413	-2.8734

Tables 3, 4, and 5 display the results of the quantile unit root tests conducted on the weekly, monthly, and quarterly frequencies of the BIST100 index. The findings reveal that, across all quantiles, the series exhibits the presence of a unit root for the weekly, monthly, and quarterly frequencies of the BIST100 index. This observation is consistent with the results of the Augmented Dickey-Fuller (ADF) test statistics. The ADF test statistics consistently indicate the presence of a unit root in the dataset covering the period 2003-2023 for weekly, monthly, and quarterly frequencies.

Table 5: Koenker-Xiao (2004) Quantile Unit Root Test Results (quarterly)

τ (Quantiles)	Coefficient (α_1)	Results	$t_n(\tau)$	Critical Values
0.1	1.0098	1	0.1505	-2.1582
0.2	0.9897	1	-0.247	-2.5293
0.3	0.9715	1	-1.1061	-2.4361
0.4	0.9623	1	-1.3332	-2.5912
0.5	0.9665	1	-1.3947	-2.7262
0.6	0.9602	1	-1.4543	-2.7244
0.7	0.9765	1	-0.6682	-2.3657
0.8	0.9538	1	-1.2449	-2.5363
0.9	1.0294	1	0.3413	-2.5642
QKS		1	1.4543	2.7913
ADF		1	-0.4773	-2.8981

*Note: * indicate 5% significance level.*

Table 6: Koenker-Xiao (2004) Quantile Unit Root Test Results (annual)

τ (Quantiles)	Coefficient (α_1)	Results	$t_n(\tau)$	Critical Values (5%)
0.1	1.7759	0	-3.9763**	-2.1200
0.2	1.3871	1	2.4879	-2.1714
0.3	1.3423	1	1.6594	-2.1200
0.4	1.4961	1	2.6142	-2.2774
0.5	1.0828	1	0.4413	-2.2122
0.6	1.0883	1	0.3226	-2.342
0.7	1.1175	1	0.6050	-2.2455
0.8	1.0349	1	0.1682	-2.5438
0.9	1.5645	0	-4.3457**	-2.5095
QKS		0	4.3457**	2.7580
ADF		1	1.587801	-3.0404

*Note: ** indicate 5% significance level.*

While the annual series shows evidence of a unit root according to the ADF test results, the quantile unit root test (Table 6) reveals that both the highest [0.9] and the lowest [0.1] quantiles are stationary. This implies that the index tends to revert to the mean in response to extreme quantiles, representing the best and worst shocks corresponding to good and bad news. It's worth noting that, due to the annual closing data neglecting numerous observations, results become more reliable as we move closer to daily frequencies.

5. Conclusions

This study investigates the dynamic structure of the Borsa Istanbul index using the linear quantile unit root test and provides insights into its long-term effectiveness. The heavy-tail distribution of the data raises concerns about the efficacy of traditional linear unit root tests, prompting the need for an alternative approach to ensure robust inference in non-normal distributions. The quantile regression method enables researchers to explore a range of conditional quantile functions, offering a more comprehensive understanding of conditional heterogeneity. Quantile unit root tests, based on quantile autoregression, have demonstrated strong performance in finite samples, as evidenced by Monte Carlo simulations that highlight substantial power gains. Particularly in the presence of a non-normal, heavy-tailed distribution, quantile unit root tests exhibit greater robustness compared to conventional OLS-based unit root tests. Hence, in this study, we apply the Koenker-Xiao (2004) linear quantile unit root test, serving as the quantile counterpart to the ADF test.

The quantile unit root tests offer a unique opportunity to scrutinize the dynamics of a series based on both the magnitude and sign of shocks. The results clearly indicate that the quantile unit root test provides more robust evidence in favour of stationarity compared to classical unit root tests. Analysing the daily frequency results reveals the presence of noticeable asymmetric dynamics. Notably, good news in the stock market exhibits a temporary, stationary process, while bad news displays a persistent, unit root behaviour. This observed asymmetry aligns with expectations for emerging market stock markets. Encompassing a broad timeframe from 2003 to 2023 and encompassing various shocks, this study recognizes the limitations of relying on a single statistical measure to summarize the entire period.

Utilizing the quantile autoregression process, quantile unit root tests allow for a nuanced understanding of asymmetric dynamics in response to both good and bad news, accounting for variations in shock magnitudes.

The results from the quantile unit root test indicate that daily data exhibits a unit root in low quantiles, suggesting that good news has a temporary effect, while high quantiles appear to be stationary, indicating persistent behaviour in response to bad news. This asymmetric dynamic reveals the nuanced nature of the stock market's reaction to different news types. In the Quantile Kolmogorov-Smirnov (QKS) test statistic, $t_n(\tau)$ is considered as the absolute supremum across all quantiles, leading to the conclusion of stationarity when compared to the critical value. Koenker-Xiao's (2004) quantile unit root test, unlike the daily stock market index, shows unit root presence across all quantiles for weekly, monthly, and quarterly frequencies. For the annual series, stationarity is observed in the highest and lowest quantiles, indicating a tendency for the extreme news deciles of that period to revert to the mean. However, it's worth noting that the annual closing data may not capture as many shocks throughout the year, making daily data a more realistic source of information.

The results clearly demonstrate distinct outcomes between the first and last quantiles, highlighting the asymmetry and magnitude of shocks. While bad news (first quantiles) exhibits persistence in the market, indicating a persistency effect, good news (last quantiles) shows a temporary impact, with the series tending to revert to the mean. This asymmetry in the response to good and bad shocks underscores the importance of examining various quantiles and deciles.

Quantile autoregression allows for a nuanced examination of the asymmetry and magnitudes in the persistence of shocks, offering insights into how positive and negative shocks influence the stock market or assets. By identifying the shocks corresponding to specific quantiles, we can assess whether the unit root behaviour changes under different economic conditions. Additionally, recognizing and understanding asymmetry becomes crucial in the context of asset pricing within the securities market.

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