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INFORMATIONAL EFFICIENCY OF THE US MARKETS FOR IMPLIED VOLATILITY BEFORE AND AFTER THE COVID-19 PANDEMIC

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Abstract

The objective of this work is to assess informational efficiency in four US markets for implied volatility. This has been pursued using daily data over 2015 to 2021 and a composite index that accounts for three possible sources of inefficiency associated with long-range dependence, short-range dependence, and entropy. The dominant pattern of long-range dependence has been that of anti-persistence both before and during the pandemic. The same applies for short-range dependence, especially before the pandemic. The presence of anti-persistence is an indication of investors' over-reaction to incoming information and implies that oscillatory trading strategies have been probably more successful that trend-following ones. During the Covid-19 pandemic, the entropy decreased in all cases suggesting that the four implied volatility series became more predictable; the intensity, however, of long-range and short-range dependence remained largely unaffected. As a result of these developments, the informational efficiency in at least two markets (those related to stock and to crude oil) fell.

Keywords: Informational efficiency, correlation structures, implied volatility, Covid-19.

JEL Classification: G14, C12

1. Introduction

The efficient market hypothesis (EMH), advanced by Fama (1965), is the cornerstone of modern Financial Economics. In its weak version, it suggests that investors are completely rational and that asset prices reflect all available past information. From a statistical viewpoint, therefore, asset prices are random walk processes, and returns are white noise processes. For Samuelson (1965), the random walk characterization may be overly restrictive; prices in informationally efficient markets are likely to be martingales. Both the random walk and the martingale characterization imply that returns do not possess any statistically significant autocorrelation structure and, as such, they are not predictable. An informationally efficient market represents a fair game pattern; no investor can expect to achieve abnormal returns systematically.

The rationality assumption in the works of Fama (1965) and Samuelson (1965) has been challenged by researchers in the field of Behavioral Finance. Shefrin (2000) and Shiller (2003) pointed out that sentiments (fear and greed) and other heuristic-driven biases influence investors' behaviour. Price changes in asset markets often occur not for fundamental reasons but because of mass psychology, instead. The Adaptive Market Hypothesis (Lo, 2004) suggests that, although market participants are mainly rational, they do sometimes make mistakes, but they learn from them, and base their predictions on trial and error. Behavioural Finance allows for discrepancies from the ideal state (informational efficiency), returns predictability, and abnormal profits even in the long term.

Given that predictability is central for informational efficiency, the economic research on the topic has largely evolved around the autocorrelation structure (intensity and pattern of serial dependence) of asset returns. The empirical literature is indeed large and it has covered stock, bond, currencies, and commodities markets (e.g. Fama, 1970; Roll, 1972; Cheung and Lai, 1995; Cajueiro and Tabak, 2004; Lim *et al*., 2008; Alvarez-Ramirez *et al*., 2008; Fernadez, 2010; Alexeev and Tapon, 2011; Chong *et al*., 2012; Kumar, 2013; Kristoufek and Vosvrda, 2014a and 2016; Mensi *et al*., 2019; Mishra, 2019, Wang and Wang, 2021; and Ftiti *et al*., 2021).

The investigation of serial dependence has relied on a large variety of statistical/econometric tools, including standard autocorrelation and integration tests, rescaled range analysis, variance ratio tests, fractal integration tests, entropy tests, detrended fluctuation analysis, wavelet transform modulus maxima and multifractal detrended fluctuation analysis. The findings vary widely depending on the time period and the market considered, as well as on the method employed.

While the autocorrelation structure of asset returns has been studied extensively, this has not been the case with their expected volatility despite the fact that the latter influences investors' decisions on portfolio optimization and risk management, and it determines how derivatives are priced. Since the early 1990s, a number of indices have been introduced to measure volatility expectations over a fixed horizon in stock and commodities markets. They are termed implied volatility indices since their value is derived/implied by the market prices of options or as "fear gauges" (Whaley, 2000) since their value is closely tied to investor sentiment (i.e., a high value of an index suggests that market participants anticipate uncertainly to rise in the near-term). In the last 15 years, markets for implied volatility have been created; futures and options for "fear gauges" are available, and investors can gain additional profit opportunities by including them in their portfolios.

Against this background, the objective of the present work is to investigate the informational efficiency of implied volatility markets in the US. To this end, it utilizes daily prices of four Chicago Board of Options Exchange (CBOE) measures, namely, the VIX (stock market), the OVX (crude oil market), the GVZ (gold market), and the EVZ (Euro-dollar exchange rate market) and a flexible approach proposed by Kristoufek and Vosvrda (2014a) that accommodates different sources of informational inefficiency and ranks markets on the base of their distance from the ideal state.

The main contributions of this work to the literature are:

(a) It considers three types of serial dependence, namely, long-memory, short-memory, and complexity. It assesses their respective contributions to the overall performance of each of the four markets for implied volatility. To the best of my knowledge, the only relevant work on the topic has been by Caporale *et al*. (2018), who examined the presence of long-run memory in the VIX series over 2014-2016.

(b) It compares autocorrelation structures both across markets as well as over time. Of particular importance here is the impact of the Covid-19 pandemic on the strength and the pattern of serial dependence, and in turn, on informational efficiency. The Covid-19 pandemic has led to a disruption of supply lines and to a decrease in aggregate demand sending unpreceded shock waves to financial markets around the globe. The recent empirical studies by Mensi *et al*. (2020), Aslam *et al*. (2020), and Choi *et al*. (2021) suggest that, as a result, the correlation structures of return series in equity and commodity markets have changed. It appears, however, that there has been no published work on the impact of the Covid-19 pandemic on the informational efficiency of implied volatility markets.

(c) It assesses the validity of a large number of individual and joint hypotheses using formal statistical tests. Earlier works on informational efficiency typically, present several statistics but draw conclusions using visual inspection only. There is no way, therefore, to tell whether the reported departures from the ideal state genuine features of the markets are considered or just the outcome of noise in the data.

In what follows, section 2 presents the analytical framework, section 3 the data and the empirical models, and section 4 the empirical results. Section 5 offers conclusions and suggestions for future research.

2. Analytical Framework

Let M_i $(i=1,2,...,n)$ be a bounded measure of informational efficiency.
Kristoufek and Vosvrda (2014a and 2014b) proposed the following composite efficiency index

$$EI = \sum_{i=1}^{n}\left(\frac{\hat{M}_i - M_i^*}{R_i}\right)^2 \quad (1),$$

where $\hat{M}_i$ is an estimate of M_i, M_i^* is the expected value of M_i under market efficiency, and R_i is the theoretical range of M_i. *EI* is, therefore, based on the distance between the actual market situation and the ideal state. Here, in line with the objectives of our work and the earlier studies of Kristoufek and Vosvrda (2014a, 2014b and 2016) and Kristoufek (2018), we consider three measures of informational efficiency, namely, the Hurst Exponent (*H*), the fractal dimension (*D*), and the approximate entropy (*AE*).

The Hurst (1951) exponent captures the long-run correlation structure (global/long-range dependence) of a time series. It takes values in [0, 1) (therefore, $R_H = 1$). When $M_H = 0.5$ the process has no long-memory (no long-range dependence); when $M_H > 0.5$, the process is persistent (i.e., it changes sign less frequently than an uncorrelated one); when $M_H < 0.5$ it is anti-persistent (i.e., it changes sign more frequently than an uncorrelated one). The fractal dimension (Mandelbrot, 1967) reflects the roughness of a stochastic process, and it can be seen as a measure of local (short-run) memory. It takes values in (1, 2] (therefore, $R_D = 1$). When $M_D = 1.5$, the process is locally uncorrelated; when $M_D < 1.5$, the process exhibits persistence (i.e., a positive (negative) change is more likely to be followed by a change of the same sign in the next non-overlapping time interval); then $M_D > 1.5$, it exhibits anti-persistence (i.e., a positive (negative) change is more likely to be followed by a change of opposite sign in the next non-overlapping time interval). A low fractal dimension signifies a low level of roughness, and it is associated with the presence of short-run trends (a black noise process) whereas a high fractal dimension signifies a high level of roughness and it is associated with the presence of short-run bursts in volatility (a pink noise process). It should be noted that, without further assumptions about the process, the long- and the short-run correlation structures are independent of each other. For a self-affine process, however, is the case that, $D = 2 - H$ suggesting that the local correlation structure is reflected perfectly in the global one[1]. Whether a time series is a self-affine process or not is empirical issue; H and *D* may, therefore, offer different insights

[1] A self-affine process is invariant in distribution under suitable scaling of time or space (e.g., Kunsch, 1987). A time series, in particular, is self-affine if it behaves the same when viewed at different time scales.

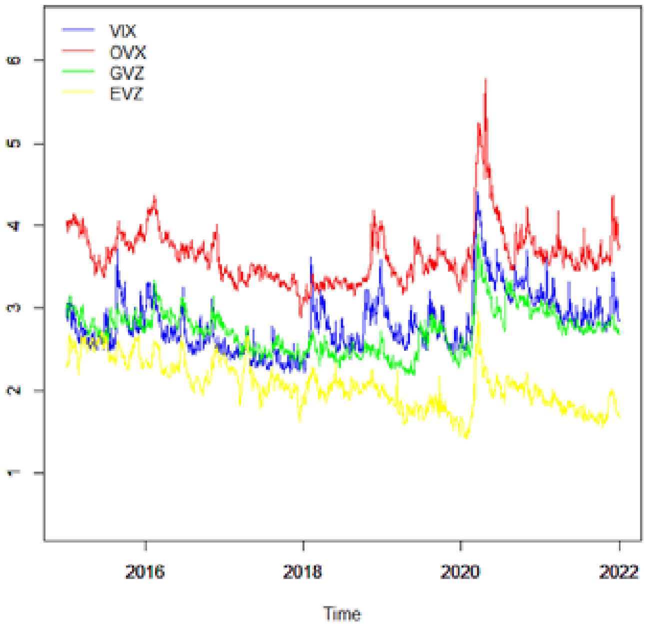
about the dynamics of a time series and it is worth investigating them separately (e.g. Kunsch, 1987; Kristoufek and Vosvrda, 2013).

Entropy is a measure of complexity. Processes with high entropy involve substantial randomness (uncertainty) and provide little information while those with low entropy can be seen as deterministic. In informationally efficient markets, prices exhibit maximum entropy. The approximate entropy (Pincus, 1991) takes values between 0 (deterministic process) and 1 (ideal state regarding informational efficiency). To ensure that, in the calculation of *EI*, all three measures have the same maximum distance (0.5) from their respective ideal states, Kristoufek and Vosvrda (2014a and 2014b) and Kristoufek (2018) suggested a rescaling of the AE range to $R_{AE} = 2$. Their suggestion has been adopted in this study as well[2].

3. The Data and the Empirical Models

The data for the empirical analysis are daily prices of the VIX, the OVX, the GVZ, and the EVZ over 1/1/2015 to 12/31/2021[3]. The VIX is the premier measure of 30-day expected volatility in the US stock market. It is calculated using the mid-point of real-time bid and ask quotes on the S&P500 index options. The OVX, is the relevant forward-looking measure of volatility for the US crude oil market and it is obtained by applying the VIX methodology to the United States Oil Fund (USO) options. The GVZ measures uncertainty in the US gold market and it is obtained by applying the VIX methodology to options traded on the Standard and Poor's Depository Receipts (SPDR) Gold Shares. The EVZ is the fear gauge for the Euro-Dollar exchange rate, and it is obtained applying again the VIX methodology to options traded on the CurrencyShares Euro Trust.

Figure 1: The evolution of fear gauge indices over 2015 to 2021



[2] Simple linear autocorrelation measures are overly restrictive for efficiency analysis since they assume that: (a) the association between current and past values is a linear one; and (b) serial dependence is a global feature of a time series (as such, they are not suitable to distinguish between long-and short-run dependence or to account for other sources of autocorrelation such as (the lack) of entropy).

[3] Obtained from *yahoo finance*.

Figure 1 presents the evolution of the four implied volatility indices from January 2015 to December 2021. The OVX and the EVZ showed downwards trends until February of 2020 whereas the VIZ and the GVZ fluctuated about their means without any visible tendency to increase or decrease. In March 2020, there was a jump in the value of all four "fear gauge" indices reflecting the uncertainty created in equity and commodities markets by the Covid-19 pandemic. Although there was a clear tendency for correction after May 2020, the implied volatility levels remained higher than those in the last months of 2019s and the first two months of 2020.

The implied volatility series have been used to compute price log-returns as $r_{it} = \ln(p_{it} / p_{it-1})$, where *i*=VIX, OVX, GVZ, and EVZ. Table A.1 in the Appendix presents summary statistics and tests on their distributions. The VIX returns appeared to be the most volatile while those of the GVZ the least volatile. All log-returns series exhibited positive and statistically significant skewness (pointing to the presence of a few large positive shocks) and excess kurtosis (pointing to leptokurtic empirical distributions). The null of normality is strongly rejected in all cases. Table A.2 in the Appendix shows the results of unit root tests on price levels and on price returns. The null hypothesis that price levels are (weakly) stationary has been very strongly rejected at any reasonable level of significance. The null hypothesis, however, that price returns are (weakly) stationary is consistent with the data. Given these findings, and following the standard practice in earlier empirical works (e.g., Kristoufek and Vosvrda, 2014a and 2016; Mensi *et al*., 2019; Wang and Wang, 2021; and Ftiti *et al*., 2021), the investigation here employs price returns.

Table 1: Tests on the departure of the individual measures and of the composite efficiency index from the respective expected values in an informationally efficient market (1/1/2015 to 3/11/2020)

Measure	VIX	OVX	GVZ	EVZ
Long Range Dependence	-0.166 (<0.01)	-0.052 (0.312)	-0.174 (<0.01)	-0.139 (<0.01)
Fractal Dimension	0.127 (0.019	0.091 (0.302)	0.273 (<0.01)	0.169 (<0.01)
Entropy	-0.203 (<0.01)	-0.247 (<0.01)	-0.264 (<0.01)	-0.232 (<0.01)
Composite Efficiency Index	0.232 (<0.01)	0.162 (<0.01)	0.352 (<0.01)	0.246 (<0.01)

Note: *The null hypothesis for long range dependence and for fractal dimension is that the value of the measure is equal to 0.5; for entropy, it is that the value is equal to 1 while for the composite efficiency index it is that it is equal to 0. The statistics are departures of the sample estimates from the respective ideal states; p-values in parentheses, obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

For the purpose of the empirical analysis, the total sample has been split in two parts, namely, from 1/1/2015 to 3/11/2020 and from 3/12/2020 to 12/31/2021. On March 11, 2020, the WHO declared the Covid-19 outbreak as a pandemic and urged countries to take immediate actions to detect, treat, and reduce transmissions in order to save people's lives. On March 12, the Dow Jones Industrial Average lost 10 percent and on March 16, it lost 12.5 percent (the fifth and the third, respectively, largest drops ever). These developments are now commonly known as the March 2020 stock market crash (e.g., Masur *et al*., 2021; Wang *et al*., 2021). The before Covid-19 pandemic sample consists of 1304 and the during the Covid-19 one consists of 458 observations.[4] All calculations are carried out in *R*. In particular, the fractal dimension has been estimated using the package Fractaldim (Sevcikova

[4] March 12 has been also selected as the starting date of the post-Covid-19 period by Zhang and Wang (2021) in their study on the impact of the pandemic on commodities futures volatility.

et al., 2021); the Hurst exponent using the package nonlinearTseries (Garcia, 2021); and the approximate entropy using the package TSEntropies (Tomcala, 2018).

The individual and joint hypotheses tests have been conducted using a Wald-type statistic

$$\Omega = (\Pi\hat{C})'(\Pi\hat{V}_C\Pi')^{-1}(\Pi\hat{C}) \tag{2}$$

where P is the restrictions' vector, C is the parameters' vector, and $\hat{V}_C$ is the bootstrap estimate of their variance-covariance matrix (Patton, 2013). Under a null, Ω follows the c^2 distribution with degrees of freedom equal to the number of restrictions.

4. The Empirical Results

Table 1 presents tests on the departure of the individual measures and of the composite efficiency index from their respective expected values in an informationally efficient market over 1/1/2015 to 3/11/2020 (before the Covid-19 pandemic). All statistics related to long-large dependence are negative; three of them (for the VIX, the GVZ, and the EVZ) are statistically significant at the 1 percent level or less while that for the OVX is not statistically significant at the conventional levels. There is evidence, therefore, that the VIX, the GVZ, and the EVZ exhibited global anti-persistence whereas the OVX had no long-run memory. All statistics related to fractal dimension are positive; three of them (for the VIX, the GVZ, and the EVZ) are statistically significant at the 2.5 percent level or less while that for the OVX is not significant at the conventional levels. There is evidence, therefore, that the VIX, the GVZ, and the EVZ exhibited local anti-persistence whereas the OVX had no short-run memory. With regard to the VIX, our results for period before the Covid-19 pandemic are in line with those of Caporale *et al*. (2018) who found that in "normal" (i.e., no-crisis) periods the implied volatility measure for the equity market showed anti-persistence. It is interesting that the local correlation structure is reflected into the global correlation one suggesting that all four process were likely to be (approximately) self-affine. The finding is consistent with what was reported by Kristoufek and Vosvrda, (2013) from their analysis of 41 stock indices. Kristoufek and Vosvrda, (2014a), however, found a positive (i.e., a non-standard) relationship between the fractal dimension and the Hurst exponent in their study of 25 commodities futures prices; in particular, Kristoufek and Vosvrda, (2014a) concluded that commodities futures price were likely to show short-run anti-persistence and long-run persistence. All statistics related to complexity are negative and statistically significant at the 1 percent level or less suggesting that none of the four-time series exhibited maximum entropy. Finally, the null hypothesis that the composite efficiency (*EI*) index is equal to 0 is rejected everywhere at the 1 percent level of less confirming, thus, the existence of informational inefficiency.

In an attempt to rank the four markets in terms of long-memory, short-memory, complexity, and composite efficiency, Table 2 presents a number of joint tests. The null hypothesis that the Hurst parameter has been the same across all markets is not rejected. This holds for the fractal dimension as well. The null hypothesis, however, the approximate entropy has been the same is rejected at the 1 percent level or less. Based on the statistics shown in Table 1, one may conclude that the least complex time series was the GVZ and the most complex was the VIX. The null hypothesis that the composite efficiency index has been the same is also rejected at the 1 percent level or less. Based on the statistics shown in Table 1, one may conclude that the least efficient was the market for the implied volatility of gold prices and the most efficient was that of crude oil prices.

Table 2: Joint tests on the individual measures and on the composite efficiency index (1/1/2015 to 3/11/2020)

Null Hypothesis	*p*-value
Long-range dependence is equal across all 4 markets	0.105
Fractal Dimension is equal across all 4 markets	0.145
Entropy is equal across all 4 markets	<0.01
Composite Efficiency is equal across all 4 markets	<0.01

Note: *The p-values have been obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

Figure 2 shows the contributions (shares) of global, local, and complex correlations to the composite index of inefficiency in the period before the Covid-19 pandemic. For the VIX, the biggest contribution came from long-range dependence, for the OVX from entropy, and for the GVZ and the EVZ from the fractal dimension. In the study of Kristoufek and Vosvrda, (2014a) the dominant source of informational efficiency was entropy while in that of Kristoufek and Vosvrda, (2016) on gold and currencies it was long-range dependence. Kristoufek (2018) found that complex correlations played a minor role in the informational efficiency of bitcoin markets relative to global and local correlations. Table 3 presents the results of joint tests where the null hypothesis is that the share of each source of inefficiency has been the same across all markets. All these nulls are rejected at the 1 percent level of less.

Figure 2: The contributions of different types of correlations to the composite efficiency index (1/1/2015 to 3/11/2020)

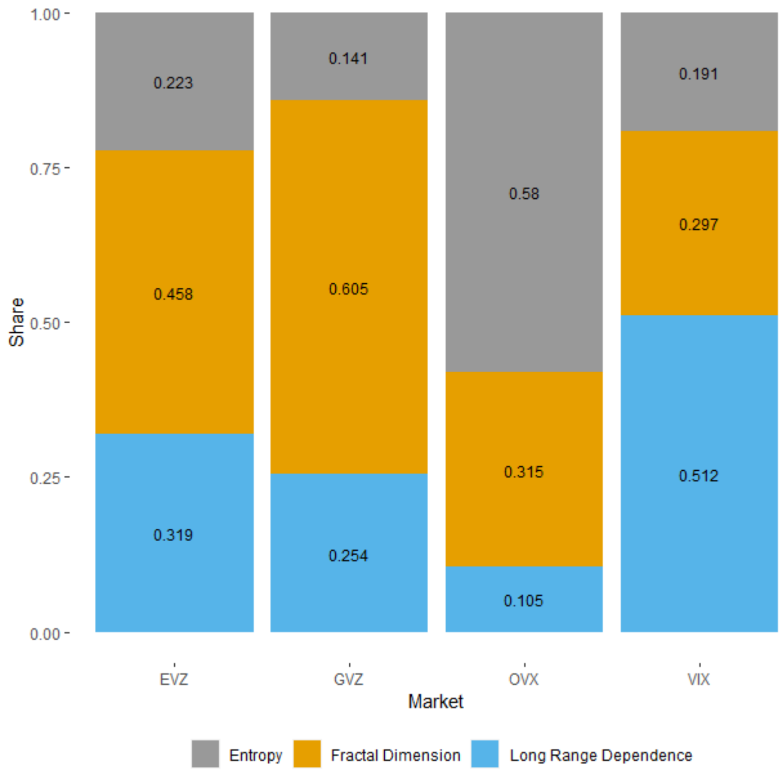


Table 3: Joint tests on (contributions) shares (1/1/2015 to 3/11/2020)

Null Hypothesis	*p*-value
The share of Long-Range Dependence is equal across all 4 markets	<0.01
The share of Fractal Dimension is equal across all 4 markets	<0.01
The share of Entropy is equal across all 4 markets	<0.01

Note: *The p-values have been obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

Table 4 presents tests on the departure of the individual measures and of the composite efficiency index from their respective expected values in an informationally efficient market over 3/12/2021 to 12/31/2021 (during Covid-19 pandemic). All statistics related to long-large dependence are negative and statistically significant at the 2.5 percent level or less providing evidence of global anti-persistence. All statistics related to fractal dimension are positive pointing again to local anti-persistence (pink noise). Only the one for the VIX, however, is statistically significant at the conventional levels. All statistics related to entropy and to composite efficiency are statistically significant at the 1 percent level or less. Table 5 presents tests on the equality of measures. The null hypotheses that the Hurst exponent and the fractal dimension have been equal across all markets are both not rejected at the conventional levels of significance. The null hypotheses that entropy and composite efficiency have been equal across all markets are rejected and the 1 and the 2.5 percent levels, respectively. Figure 3 shows the contributions of global, local, and complex correlations to the composite index of inefficiency during the Covid-19 pandemic. For the VIX, the biggest contribution came from the fractal dimension, for the OVX from long-memory, and for the GVZ and EVZ from entropy. Table 6 presents the results of joint tests where the null hypothesis is that the share of each source has been the same across all markets. All these nulls are rejected at the 1 percent level of less.

Figure 3: The contributions of different types of correlations to the composite efficiency index (3/12/2020 to 12/31/2021)

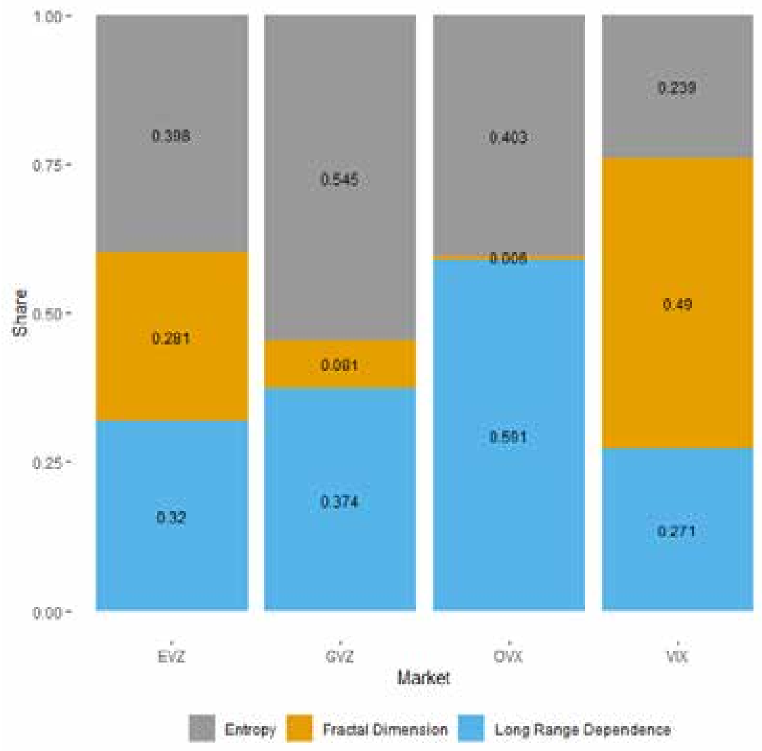


Table 4: Tests on the departure of the individual measures and of the composite efficiency index from the respective expected values in an informationally efficient market (12/3/2020 to 12/31/2021)

Measure	VIX	OVX	GVZ	EVZ
Long Range Dependence	-0.192 (<0.01)	-0.225 (<0.01)	-0.182 (<0.01)	-0.181 (0.012)
Fractal Dimension	0.258 (<0.01)	0.024 (0.852)	0.085 (0.359)	0.169 (0.142)
Entropy	-0.361 (<0.01)	-0.371 (<0.01)	-0.439 (<0.01)	-0.404 (<0.01)
Composite Efficiency Index	0.369 (<0.01)	0.293 (<0.01)	0.298 (<0.01)	0.320 (<0.01)

Note: *The null hypothesis for long range dependence and for fractal dimension is that the value of the measure is equal to 0.5; for entropy, it is that the value is equal to 1 while for the composite efficiency index it is equal to 0. The statistics are departures of the sample estimates from the respective ideal states; p-values in parentheses, obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

The visual comparison of Table 1 and Table 4 suggests that there are differences between the values of the correlation measures and of the composite efficiency indices before and during the Covid-19 pandemic. To verify whether these represent actual changes in the correlation structures and in the level of informational efficiency or they are just the outcome of noise in the data, Table 7 shows the results of formal equality tests. The relevant statistics are values during minus values before. The statistics related to long-range dependence all negative; only for the OVX, however, the difference is statistically significant (at the 10 percent level). One may conclude that, with a possible exception the implied volatility measure for the crude oil market (which turned out to be more anti-persistent), the Covid-19 pandemic did not have any strong influence on the long-memory of the implied volatility series. Two of the statistics related to fractal dimension (for the OXV and the GVZ) are negative, one (for the VIX) is positive and one (for the EVZ) is zero; only for the GVZ, however, the difference is statistically significant (at the 10 percent level). One may conclude that, with a possible exception the implied volatility measure for the gold market (which turned out to be less anti-persistent), the Covid-19 pandemic did not have any strong impact on the short-run correlations of the “fear gauge” indices. The statistics related to entropy are all negative and statistically significant at the 1 percent level or less providing evidence that the complexity of the implied volatility series decreased during the Covid-19 pandemic. Three of the statistics related to the composite efficiency index (for the VIX, the OVX, and the EVZ) are positive and one (for the GVZ) is negative. Also, the statistics for the OVX and the VIX are statistically significant at the 5 and the 10 percent level, respectively. It appears that informational efficiency has decreased for the crude oil (and possibly for the equity) implied volatility markets whereas it remained the same for the gold and the currency markets.

Table 5: Joint tests on the individual measures and on the composite efficiency index (3/12/2020 to 12/31/2021)

Null Hypothesis	*p*-value
Long Range Dependence is equal across all 4 markets	0.966
Fractal Dimension is equal across all 4 markets	0.439
Entropy is equal across all 4 markets	0.01
Composite Efficiency is equal across all 4 markets	0.025

Note: *The p-values have been obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

Table 6: Joint tests on (contribution) shares (12/3/2020 to 12/31/2021)

Null Hypothesis	*p*-value
The share of Long Range Dependence is equal across all 4 markets	<0.01
The share of Fractal Dimension is equal across all 4 markets	<0.01
The share of Entropy is equal across all 4 markets	<0.01

Note: *The p-values have been obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

Table 7: Tests on the equality of the individual measures and of the composite efficiency index before and during the Covid-19 pandemic (1/1/2015 to 3/11/2020 vs 3/12/2020 to 12/31/2021)

Null Hypothesis	VIX	OVX	GVZ	EVZ
Long Range Dependence is the same	-0.026 (0.747)	-0.172 (0.078)	-0.005 (0.957)	-0.024 (0.622)
Fractal Dimension is the same	0.132 (0.236)	-0.067 (0.671)	-0.189 (0.071)	0 (0.982)
Entropy is the same	-0.157 (<0.01)	-0.124 (<0.01)	-0.175 (<0.01)	-0.171 (<0.01)
Composite Efficiency is the same	0.137 (0.064)	0.13 (0.046)	-0.054 (0.419)	0.074 (0.264)

Note: *The statistics are values during minus values before the Covid19 pandemic; p-values in parentheses, obtained using block-bootstrap (Politis and Romano, 1994) with 5000 replications.*

Choi (2021), using Multifractal Detrended Fluctuation Analysis (MFDFA), concluded that stock markets returns in the US (sectors that are parts of the S&P 500 index) become more persistent since the Covid-19 outbreak; Ozkan (2021), using variance ratio tests, found that the stock markets of six developed countries (US, UK, France, Germany, Italy, and Spain) became less efficient during the pandemic; Aslam *et al*. (2020), using data from the forex markets and MFDFA, concluded that global persistence increased since the Covid-19 outbreak; and Mensi *et al*. (2020), using data from crude oil and gold markets and MFDFA, reported that the respective returns series became anti-persistent during the pandemic. Although the findings of these very recent studies are not directly comparable to ours, they do offer support for the hypothesis that the Covid-19 pandemic has probably deteriorated the performance of certain markets.

Table 8: Tests on the equality of contributions (shares) shares before and during the Covid-19 pandemic (1/1/2015 to 3/11/2020 vs 3/12/2020 to 12/31/2021)

Null Hypothesis	VIX	OVX	GVZ	EVZ
The share of Long Range Dependence is the same	-0.241 (0.097)	0.485 (<0.01)	0.119 (0.293)	-0.001 (0.992)
The share of Fractal Dimension is the same	0.193 (0.445)	-0.308 (0.304)	-0.523 (0.025)	-0.178 (0.543)
The share of Entropy is the same	0.047 (0.832)	-0.177 (0.540)	0.404 (0.064)	0.175 (0.513)

Note: *The statistics are shares during minus shares before the Covid19 pandemic; p-values in parentheses, obtained using block-bootstrap (Politis and Romano, 1994) with 1000 replications.*

The visual inspection of Figures 2 and 3 indicates several sizable changes in the contributions of global, local, and complex correlations to the composite index of informational efficiency. Nevertheless, from Table 8 (which presents the results of formal equality tests) it follows that, out of 12 differences, only 2 are statistically significant and the 2.5 percent level or less and 2 more at the 10 percent level or less. For the OVX (GVZ) the share of long-range dependence (fractal dimension) has increased; for the GVZ the share of entropy has increased and whereas for the VIX the share of global correlations has decreased. Overall, it appears that the impact of Covid-19 pandemic on the relative importance of different sources of inefficiency to the performance of the implied volatility markets was limited.

The evolution of individual implied volatility indices and, thus, the autocorrelation structures of respective time series along with the extent and the composition of inefficiency depend on investors' perceptions about future uncertainties. As far as crude oil is concerned, and prior to Covid-19 pandemic, the main preoccupation of oil traders had been sudden price downswings as a result of the shale oil revolution. In the Euro-USD market, investors typically tend to fear a sudden drop of the Euro relative to USD[5]. The monetary policy exercises a key influence on the evolution of the GVZ series (e.g., Norland, 2019). In a normal monetary environment, where interest rates are well above zero, gold traders are more concerned with rising than with falling prices while the opposite is the case with near- zero interest rates. From 2015 to 2021, there were periods of quantitative tightening (2017-2019) and easing (after the Covid-19 outbreak). At the same time, gold is a safe-haven asset and the GVZ captures (part of) the general economic uncertainty (e.g., Pandungsaksawadi and Daigler, 2014). The finding in Table 1, for example, that prior to Covid-19 pandemic the GVZ market was less efficient than the OVX market may imply that indices reflecting fear across multiple asset markets (as the GVZ does) exhibit stronger serial correlation than asset-specific ones (such as the OVX). It is noteworthy that during the Covid-19 pandemic, where perceptions of fear across all asset classes have been aligned, the dispersion of inefficiency levels turned out to be much smaller relative to the immediately preceding period (Table 4).

5. Conclusions

The objective of this work has been to investigate the informational efficiency of implied volatility markets in the USA. To this end, measures of long-memory, fractal dimension, and complexity for four "fear gauge" indices have been estimated and employed as inputs to evaluate the performance of markets related to equity, crude oil, gold, and currencies. The empirical findings suggest:

(a) The local and the global serial dependence structures have been similar both across markets as well as before and during the Covid-19 pandemic. A possible explanation lies in the existence of uncertainty spillovers. The relevant literature (e.g., Badshah *et al*. 2013a; Liu *et al*., 2013; Lowen *et al*., 2021) has pointed to a number of direct and indirect transmission channels among the implied volatility markets including the flight-to-safety effect, the impact of exchange rate volatility on firms that are not fully hedged, and the financialization of commodities.

(b) The dominant pattern of long-range dependence both before and during the pandemic has been that of anti-persistence. The same applies, especially for the period before the pandemic, for local dependence. The presence of global dependence implies that the interaction between supply and demand (arbitrage) has not eliminated opportunities for abnormal profits even over longer horizons. This is consistent with the notion that sentiment (fear or exuberance) may have a lasting influence on investor behavior. Furthermore, as noted by Fernadez (2010), anti-persistence indicates that participants in financial markets tend to over-react to incoming information. This, in

[5] https://www.risk.net/derivatives/currency-derivatives/6553576/fx-options-skews-economics-and-implications.

turn, suggests that oscillatory trading strategies have been more likely to “beat” the markets relative to trend-following ones.

(c) The entropy of all series has decreased during the pandemic; in other words, the VIX, the OVX, the GVZ, and the EVZ have become more predictable relative to the immediately preceding period. Given, that “fear gauge” indices are forward-looking measures of uncertainly, the decrease in entropy is probably a reflection of the markets’ opinion that, the one-of-a-kind crisis triggered by the Covid-19 outbreak, had been very likely to increase price uncertainly and push the measures of implied volatility systematically in one direction (upwards).

(d) The complexity measures have been different both before and after the Covid-19 pandemic. This (given the similarity of global and local correlation structures) implies that the reduction in entropy has been the main cause of the deterioration in the performance of (at least two) markets for the implied volatility in the USA. The decrease in informational efficiency is consistent with the predictions of Behavioral Finance (e.g., Badshah, 2013b; Low, 2004) that crises, by reinforcing the role of sentiment and by placing time pressure on investors to use rules of thumb or short-cuts, may increase the likelihood of incorrect judgments (mispricing).

(e) The contributions of long-range dependence, short-range dependence, and entropy on the composite efficiency index differ across markets. The Covid-19 pandemic, however, has had a limited impact on the relative importance of different sources of inefficiency on the overall performance of the implied volatility markets.

There are a number of avenues for future research. One may involve a finer analysis of the correlation structures by allowing for different serial dependence patterns under positive and negative changes. Another may investigate potential changes in the intensity of spillovers among the “fear gauge” indices during the Covid-19 pandemic. In any case, additional work on this elaborate topic is certainly warranted.

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Appendix

Table A.1. Descriptive statistics and tests on the distributions of price log-returns

Statistic	VIX	OXV	GVZ	EVZ
Mean	0.000	-0.001	-0.002	-0.004
Median	-0.007	-0.004	-0.004	0.000
SD	0.084	0.066	0.052	0.057
Minimum	-0.299	-0.622	-0.266	-0.402
Maximum	0.768	0.858	0.297	0.496
1st Quartile	-0.048	-0.031	-0.03	-0.029
3rd Quartile	0.037	0.027	0.025	0.027
Skewness	1.135	1.949	0.578	0.235
	(<0.01)	(<0.01)	(<0.01)	(<0.01)
Kurtosis	10.314	33.413	6.276	13.636
	(<0.01)	(<0.01)	(<0.01)	(<0.01)
Normality	0.921	0.818	0.956	0.903
	(<0.01)	(<0.01)	(<0.01)	(<0.01)

Note: *The p-values for skewness, kurtosis, and normality have been obtained using the tests by d'Agostino (1970), Anscombe and Glynn (1983), and Shapiro and Wilks (1965), respectively.*

Table A.2. Unit root tests

With	ln(VIX)	ln(OVX)	ln(GVZ)	ln(EVZ)
Constant	1.922	0.621	0.973	4.562
Trend	0.478	0.600	0.934	0.406
	dln(VIX)	**dln(OVX)**	**dln(GVZ)**	**dln(EVZ)**
Constant	0.019	0.035	0.036	0.014
Trend	0.018	0.021	0.025	0.014

Note: *The critical values for the KPSS (Kwiatkowski et al., 1992) test with a constant are 0.347, 0.436, and 0.739 and with a deterministic trend are 0.119, 0.146, and 0.216 at the 10, the 5, and the 1 percent level, respectively.*