Market conditions and time varying conditional correlations

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This paper shows how the dependency of time-varying conditional cross-correlation on prevailing market conditions can be modeled. With this modelling approach it is possible to empirically investigate how correlations between different markets are dependent on market volatilities and other external factors. The paper shows that the modeling of conditional correlations can be simplified to modeling a univariate GARCH-process. The advantage of this approach is that it allows for utilization of existing GARCH methodology and software to estimate the dynamics of correlation. Our empirical results reveal that time-varying correlations between stock markets are largely unpredictable and are at best dependent on world market volatilities. Weaker evidence is found that correlations are driven by market downturns.

Keywords: Conditional correlation, Volatility, GARCH

1. Introduction

Numerous empirical studies document that contemporaneous correlations between international stock markets' returns are unstable over time. In this regard, it is well known that correlations among international markets tend to increase when stock returns fall precipitously. Although the need for better understanding of asset correlations and their evolution over time is widely acknowledged (e.g., see Pesaran and Pesaran (2010)), only a few studies have sought to examine the sources of time-varying correlation. Some results suggest that volatility is a major driver of correlation. However, applying extreme value theory, Longin and Solnik (2001) argue that correlation is not related to market volatility but to the market trend. Using monthly data, they find that, especially in the case of negative returns in international markets, correlations tend to increase. Consequently, they infer that correlation tends to increase in bear markets but not in bull markets. Also, they infer that conditional correlation is mainly affected by market trend rather than volatility in periods of extreme returns. Thus, the limited empirical evidence on the question of what drives time-varying correlation is mixed.

In this paper we attempt to contribute further evidence on the sources of correlation between stock returns. Unlike the aforementioned studies that examine changing correlation across either various sub-periods, conditional or extreme value correlation, or correlation under different volatility regimes, we model the dependence of the correlation *directly* on the level of prevailing uncertainty, which is measured in terms of volatilities and other external factors. As a first step we model the individual time-varying volatilities and remove the impact of the time-varying volatilities on the time series by standardization. As a result we have unit variance homogenous series and any time-varying behavior in the covariance must stem from varying dependency. With this approach we can straightforwardly model any bivariate correlation using standard GARCH modeling and existing software. Importantly, this modeling approach enables us to comparatively investigate how time-varying correlation is affected by local volatilities, global market volatility, and other factors such as market trend as well as to evaluate in the same model the relative importance of these two major sources (trend and volatility) on time-varying correlation.

We collect daily returns for markets in North America (United States), Asia (Japan), and Europe (United Kingdom, France, Germany, Switzerland, the Netherlands, Denmark, Sweden, Norway, and Finland), in addition to the MCSI World Index as a proxy for the world market. The analyses on markets with approximately common opening hours are run on daily bases and analyses on intercontinental markets with non-overlapping opening hours are run by utilizing weekly data to avoid technical lead-lag relationships due to different trading hours.

Consistent with some earlier studies, our preliminary analyses for the period 1990–2009 appear to suggest that volatilities and correlations are significantly higher when world markets are down trending in a bear market. However, more structured analyses using our GARCH model approach for the time-varying joint behavior reveal that this effect is secondary in importance to volatility as a driver of correlation. Generally we find that time-varying correlations between stock market returns are very weakly predictable and are primarily explained by national market volatilities that decrease correlation (negative signed coefficient) and external world market volatilities that increase (positive signed coefficient) correlation. This is the case in particular among the major European markets. Down trends in world markets are significant at times but have a relatively weaker relationship than volatility to correlations between stock market returns. We also find a day-of-the-week effect on the correlations such that on Mondays correlations between markets tend to be higher than other week days. Furthermore, it is evident that changes in correlations do not share similar clustering features and predictability as volatilities and that national market and world market volatility, if any, are candidates for the major drivers of time-varying correlation between international stock markets.

2. Methodology

In the GARCH literature, time-varying correlation is often assumed to exhibit a clustering characteristic in the same manner as volatility. This approach has the fundamental shortcoming of not seeking to identify the sources of changing volatility and changing correlation. The focus is more on a description of the process, rather than on explicit understanding what explains changes in correlation, such as the magnitude of risk or other factors. It is extremely difficult to model correlations directly as functions of exogenous variables and at the same time guarantee positive definiteness. There is no obvious solution to the latter problem. Consequently, rather than solving this problem, the present paper models the correlation in the limited bivariate case. We model the (time varying) correlation by starting with first filtering out the time varying volatilities from the series and form an index of the filtered series. If there is any time-varying volatility in the index, we can show that it must come from the time-varying correlations between the stocks in the index. The usefulness of this approach is that time-varying correlation can simply be modeled via standard GARCH.

Following Engle (2002), we write the conditional correlation of (return) series u_t and v_t as

$$\rho_t = \frac{\mathrm{cov}_t[u_t, v_t]}{\sqrt{h_{u,t} h_{v,t}}} = \mathrm{cov}_t(z_{1t}, z_{2t}) = E_{t-1}(z_{1t} z_{2t}) \quad (1)$$

where $u_t = \sqrt{h_{u,t}}\, z_{1t}$ and $v_t = \sqrt{h_{v,t}}\, z_{2t}$. Both z_{1t} and z_{2t} are (weak white noise) processes with zero mean and unit variance in which the subscript denotes the predicted entity given information up to time point $t-1$.

In the simple case when the correlation is time invariant, we see from the first part of equation (1) that the time-varying covariance must change in a fixed proportion to the product of the time-varying standard deviations. In this case asymmetries in volatilities reflect asymmetries in the covariance as well. Consequently, it may be difficult to infer on the basis of the covariance whether the dependence per se between the series is time-varying or due simply to the fixed relation between the volatilities and covariance determined by the time invariant correlation.

In order to avoid these problems, we investigate the time variability in the covariation by allowing the dependence structure itself to be time-varying (rather than a constant). We propose a direct and very simple approach to model time-varying correlation using existing GARCH facilities. This approach utilizes the same principle as the copula theory in the sense that we model in separate steps the time-varying individual volatilities (margins) and the time-varying dependency (joint behavior) between the series. We make use of the fact that $z_{it} = u_t / \sqrt{h_{i,t}} \sim (0,1)$, $i = 1, \ldots, n$, i.e., distributed with mean zero and unit variance, such that z_{it} are weak white noise processes. The white noise property of z_{it} with time-invariant means and variances implies that, if there is any time variation in the joint behavior of these series, it must be in the correlation or more generally in the dependency, due to constant (conditional) means and constant (conditional) variances. In order to model the average time-varying conditional correlation, we define the following scaled index

$$z_t = (z_{1t} + \cdots + z_{nt}) / \sqrt{n(n-1)} \approx (z_{1t} + \cdots + z_{nt})/n \,. \quad (2)$$

The first step estimates individual GARCH-processes, and the second step applies a GARCH model to a suitably scaled portfolio z_t of the standardized series such that $h_{z,t} \approx \bar{\rho}_t$, where $\bar{\rho}_t$ is the average time t contemporaneous correlation of individual series. Using the white noise properties of the standardized series and the traditional portfolio variance formula, we have

$$h_{z,t} = \frac{1}{n-1} + \bar{\rho}_t, \quad (3)$$

where $\bar{\rho}_t$ is the average cross-correlation of the standardized GARCH processes, $z_{it} = u_{it}/\sqrt{h_{it}}$. Thus, for large n the conditional variance of the scaled index of the individual series equals approximately the average conditional contemporaneous correlation.

In order to have a positive definite covariance matrix, on average the correlations must be within the range from $-1/(n-1)$ to $+1$. To derive time-estimates of the conditional correlation, a transformation that forces the estimated correlation within the limits of $(-1/(n-1),\ +1) \approx (0,1)$ can be applied. One such transformation used in these circumstances is the logit transformation. The logit transformation also suggests that an EGARCH specification would be a useful candidate in practical applications to estimate volatility $h_{z,t}$. We follow this practice and utilize the following EGARCH(1,1) specification in our empirical applications,

$$\log h_{z,t} = \omega + \alpha \tilde{z}_{t-1} + \gamma(|\tilde{z}_{t-1}| - E|\tilde{z}_{t-1}|) + \beta h_{z,t-1} + x_t'\theta, \quad (4)$$

where $\tilde{z}_{t-1} = z_{t-1}/\sqrt{h_{z,t-1}}$, and x_t is the vector of additional explanatory variables, including time trend, world market volatility, local volatilities (log of the product), a dummy variable to indicate bearish market, and daily dummies to capture possible day-of-the-week effects. For the residual in the mean equation, we assume a generalized error distribution (GED).

The role of the time trend is to capture the average effect of unknown or unobservable common factors such as market globalization that may affect the correlation. World market volatility serves the purpose of capturing the impact of market-wide risk on time-varying correlation. Persistence in volatility guarantees that, even if there is only a contemporaneous relation between volatility and correlation, one period ahead prediction can be used as a reasonable surrogate of the contemporaneous volatility due to the persistence. In the same fashion we seek to capture the prevailing market trend by using a dummy variable, which indicates down or up trends in a smoothed world index series, rather than using daily changes in the index. We smooth the world index by using an exponentially weighted moving average (EWMA) with a weight of 0.05 for the last observation and 0.95 for historical values.

In the first stage the individual volatilities are estimated via a GARCH-model. Unfortunately, there are no general guidelines on choosing among a wide variety of possible GARCH-specifications. In stock return data, however, accounting for leverage and fatter tails than implied by conditional normal distribution seem to be important factors [Hansen and Lunde (2005)]. Therefore, to accommodate these features, we model the individual series using TGARCH(1,1) with conditional t-distribution in which the degrees of freedom is a free parameter. To capture possible autocorrelation in returns, we use an appropriate ARMA-specification of the mean. Thus, the volatility of the residual return series i is modeled using the following TGARCH(1,1):

$$h_{it} = \omega + \alpha_i u_{i,t-1}^2 + \gamma_i D_{it} u_{i,t-1}^2 + \beta_i h_{i,t-1}, \quad (5)$$

where $D_{it} = 1$ if $u_{it} < 0$ and zero otherwise, and $u_{it}/\sqrt{h_{it}} \sim t(v_i)$, t-distribution with v_i degrees of freedom.

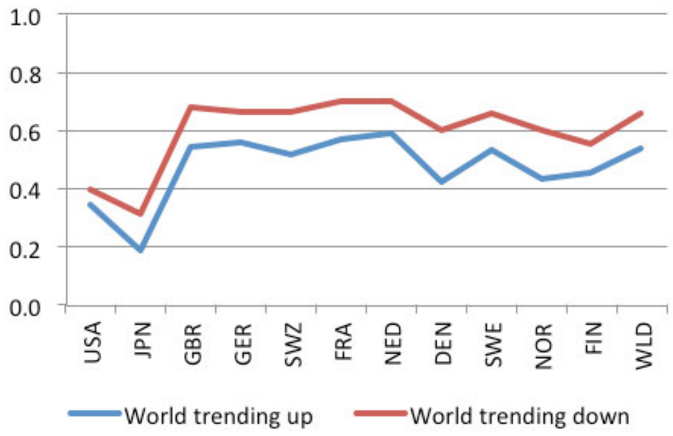
3. Data

We utilize daily close-to-close stock index returns for twelve national stock markets: United States (S&P500), Japan (Nikkei 225), United Kingdom (FT100), Germany (DAX), Switzerland (SSMI), France (CAC40), the Netherlands (AEX General), Denmark (KFX), Sweden (Stockholm General), Norway (OSE All Share), and Finland (HEX All Share), in addition to the MCSI world index (in US dollars). The sample series starts January 2, 1990 and ends August 2009. Index quotations for national holidays are deleted. Data is obtained mainly from the websites for Yahoo (http://finance.yahoo.com) and Global Financial Data Inc. (www.globalfinancialdata.com) where complete descriptions of the indices can be found.

In European markets, volatility is around 20% in annual terms. All the markets exhibit slight negative skewness and strong kurtosis, as well as highly statistically significant squared return autocorrelations (volatility clustering). Additionally, the small Nordic markets and the world market index have slight though statistically significant return autocorrelations. For the US market, autocorrelation is negative and statistically significant.

Figure 1 presents the average pairwise contemporaneous correlations with other markets in up and down trending world markets. The impact of the world market trend is measured using a world market down-dummy. World markets are considered down when the EWMA estimated trend is less than the corresponding estimate for the previous period.

Figure 1: Average pairwise correlations in up trending and down trending markets



On average we find that the correlations are 0.12 (26%) larger in the down markets than in the up markets. Furthermore, we find that volatility was on average about 62% higher in the down markets, and the difference in each case was highly statistically significant. The smallest increases in volatility during the down market were 49% for Finland and 50% for Japan, and the largest was 80% for the USA.

4. Empirical Results

We split the empirical analyses into three separate markets: major European markets, Nordic markets, and world leading markets. The former markets are UK, Germany, Switzerland, France, and Netherlands. Nordic markets are Denmark, Sweden, Norway, and Finland. The latter world leading markets are USA, Japan, UK, and Germany.

For the European markets there is no major problem with non-overlapping trading hours. For this reason we use daily returns for analyses of European markets and weekly returns for the world leading markets.

The results of the estimated ARMA-TGARCH(1,1) models for the individual markets show generally the expected characteristics. First, the ARMA-structure is statistically significant for the Nordic markets and the world market. Second, the ARCH, GARCH, and asymmetry are statistically significant at the 10% level, except for the ARCH component for USA, France, and the world index.

The results of modeling time-varying conditional correlations are summarized in Table 1. This table tabulates the number of times an explanatory variable in the model has a statistically significant coefficient with respect to explaining the variation in the conditional correlation. The columns represent the three groups of markets.

Table 1. Number of times the variable has a significant coefficient in the estimated conditional correlations (at the 5%-level).

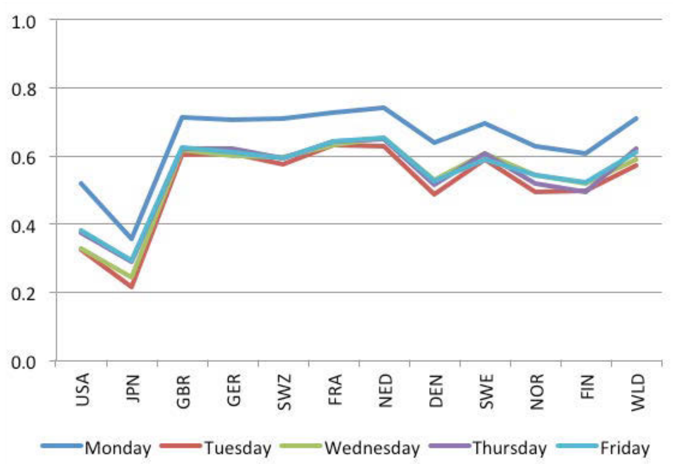
Independent variables	Major European	Nordic markets	Leading World
Constant	4	1	3
ARCH	7	0	4
GARCH	1	2	2
Asymmetry	6	6	5
Local volatilities	4	1	0
World market	5	1	1
World trend down	0	0	0
Monday	8	2	na
Tuesday	0	5	na
Wednesday	0	0	na
Thursday	0	5	na
Time trend	3	4	1
Total number	10	6	5

The empirical results for the main European markets reveal an interesting characteristic of the time-varying correlation. Unlike clustering patterns of volatilities, the estimated ARCH and GARCH parameters for the conditional correlations indicate distinctly different patterns. First, in all correlations the estimates of the ARCH term are small and negative (smaller in absolute value than 0.17). This effect is further pronounced by a negative leverage coefficient in some of the EGARCH-specifications. Second, the persistence or the GARCH parameters are typically not statistically significant. The only exceptions are the correlations of Germany/France and France/Netherlands. Thus, the general conclusions for these markets are that, unlike the case of volatilities, the dependence structure of the series' own (joint) behavior poorly predicts future correlation and the clustering phenomenon typical to volatilities is not characteristic for correlations.

The analyses of external factors explaining the time-varying correlation again show a clear pattern. In 4-out-of-10 cases the estimated coefficients for local volatilities are significant at the 5% level in two-sided testing – namely, UK/Germany, UK/France, UK/Netherlands, and Germany/France. In all cases, including insignificant ones, the coefficient estimates are negative. These results imply that the likely impact of increased local uncertainty is to reduce correlations. However, the situation is quite different for the world market volatility that proxies global volatility or uncertainty. In all cases, including insignificant ones, the coefficient estimates are positive. Thus, the results indicate that, while increases in uncertainties stemming from domestic sources tend to decrease market co-movement, uncertainties stemming from global sources tend to increase market co-movements. Interestingly, the dummy variable reflecting down trend in global markets is typically not statistically significant. Together with the much more frequent significance of the world market volatility, our results support the view that cross-correlations between markets are primarily driven by the level of global volatility (uncertainty) rather than market trends.

Day-of-the-week effects are evident in all markets, which is due to the increased correlation on Mondays in all other but UK/France and France/Netherlands correlations. The day-of-the-week effects are shown in Figure 2.

Figure 2: Average pairwise correlations on different week days.



In all estimations the GED-shape parameter is statistically significantly below 2, which indicates non-normality of the conditional distribution. Diagnostic statistics indicate no further autocorrelation in the squared standardized residuals, which suggests that the specified models capture the time-variability due to correlation in the scaled series.

We infer that national volatility and world volatility are important when analyzing changes in correlation over time and that world down trend is of lesser importance.

For the smaller Nordic markets, the characteristics of the conditional correlation are slightly different. Due to the fact that these markets are smaller, correlation is less dependent of ARCH effects and local volatilities.

For the world leading markets, using weekly data, we find generally that the estimates of the ARCH term and the asymmetry coefficient are statistically significant and negative. However, some persistence is found in the UK/Japan and Germany/Japan correlations with statistically significant GARCH parameters. Local volatilities are not found to be significant in any of the equations. The world down trend is insignificant in all equations. We infer that generally there is very little common structure in conditional correlations between leading markets that can be predicted from historical sources. Diagnostic statistics support this conclusion by indicating no remaining autocorrelation in the squared standardized residuals.

5. Conclusion

Initial empirical analyses of stock market returns using daily data confirmed the common finding that correlations are more pronounced when the world market index is trending down. This initial observation, however, received very little support in subsequent more structured analysis. We found that the major determinant of time-varying correlations between stock markets was world market volatility. Unlike the situation for volatilities, the own history of correlation had very little predictive power for future correlation. After controlling for local market volatilities, world bearish market trend, general time trend, and day-of-week effects, the single best predictor was the world market volatility. Another pattern in the correlation equations was the day-of-the-week effect, wherein markets tended to be more correlated on Mondays than other week days. This effect was less pronounced in smaller Nordic market correlations.

Our results partially support earlier studies indicating that mutual correlations tend to increase when volatility is high (Solnik, Boucrelle, and Fur (1996); Ramchand and Susmel (1998); Dennis, Mayhew, and Stivers (2005); Baele (2005) and others). However, we find little evidence that correlations between stock market returns in different countries increase during worldwide bearish markets (Longin and Solnik (2001); Ang and Chen (2002) and others).

We should emphasize that our GARCH-type analyses employ historical information to predict future correlation. In this regard, the overall conclusion is that there is low prediction power. Unlike our approach, previous studies cited above typically utilize contemporaneous time analyses. In this case, stock markets likely share the same direction but it is largely unpredictable. We leave this interesting question for future research.

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